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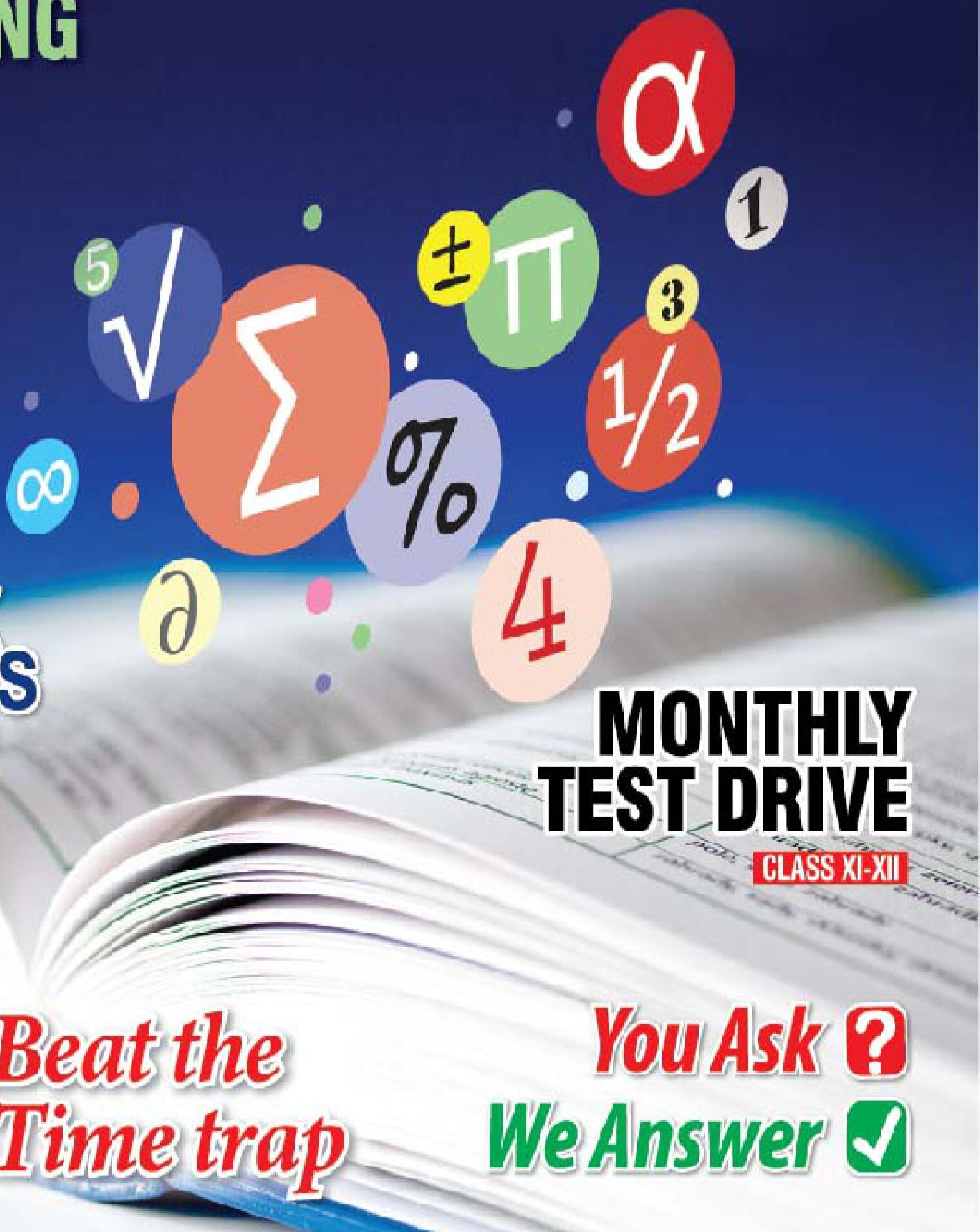

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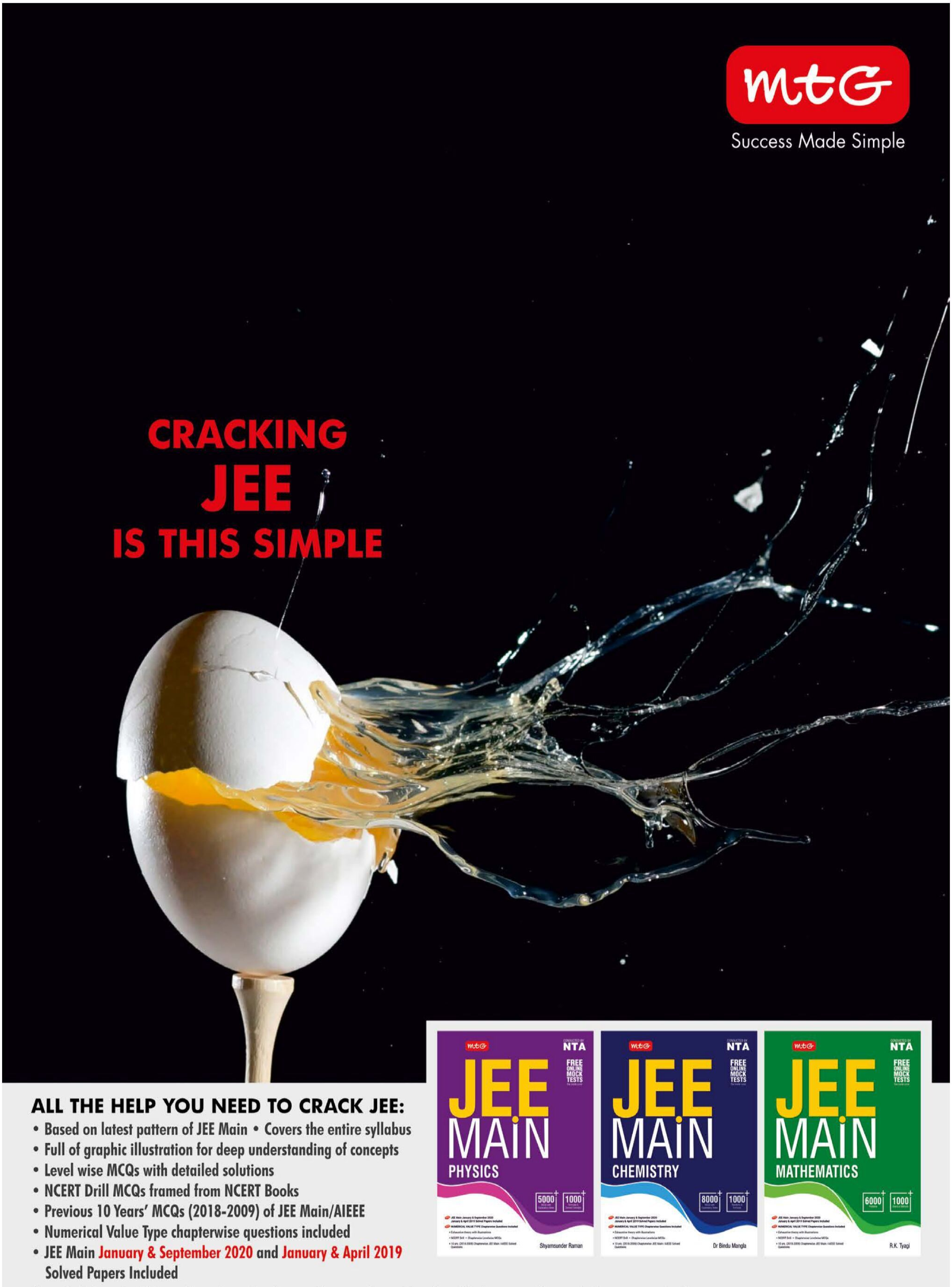
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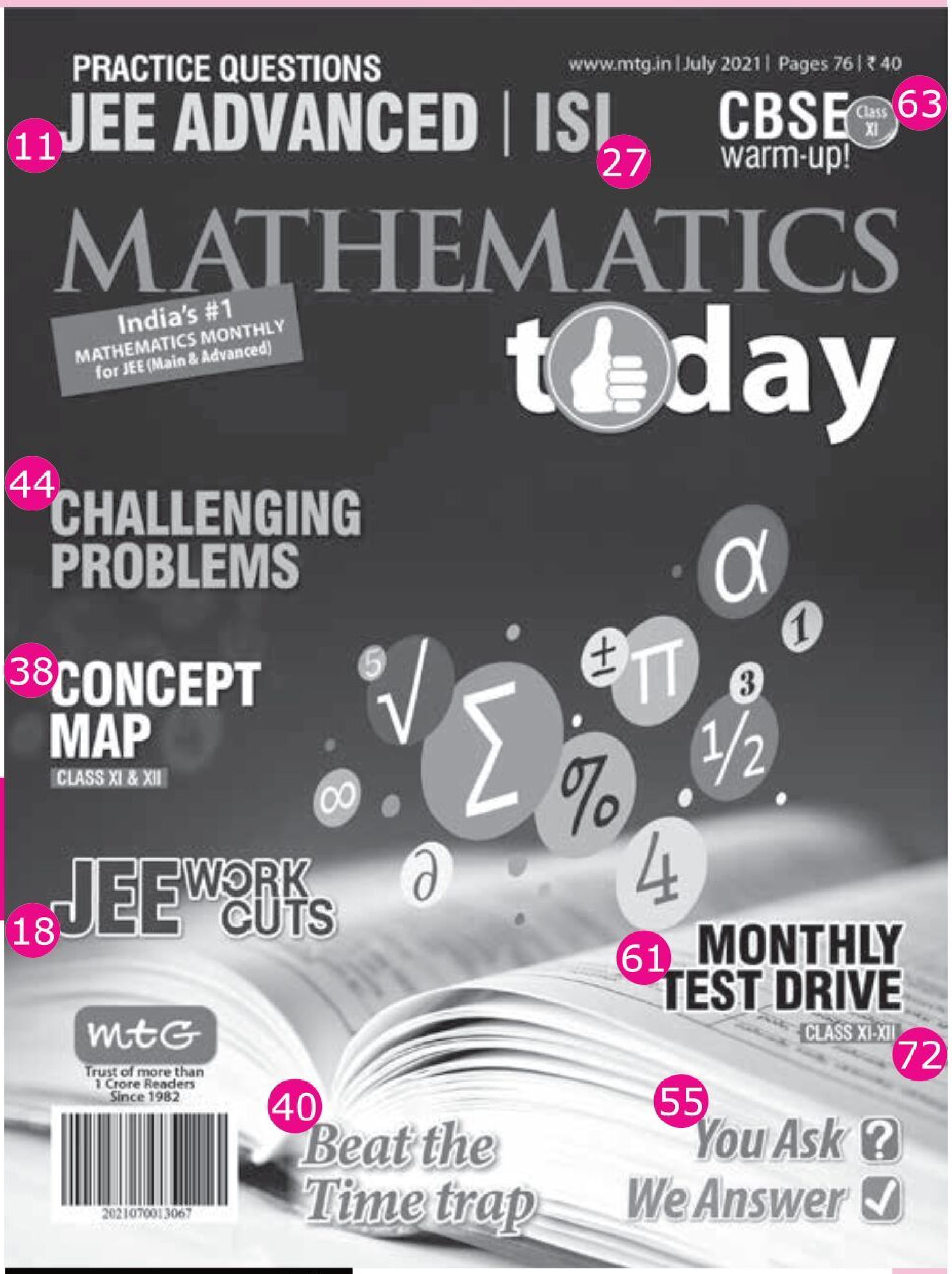
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FOCUS
JEE MAIN 2021

Section A will be of Multiple Choice Questions (MCQs). Section B will contain questions whose answers are to be filled in as a Numerical Value. In Section B candidates have to attempt any five questions out of 10.

SECTION A (MULTIPLE CHOICE QUESTIONS)

1. If $f: R \to R$ is defined by $f(x) = x - [x] - \frac{1}{2}\ \forall\, x \in R$, where $[x]$ denotes the greatest integer function, then solution set of $f(x) = 1/2$ is given by
(a) Z, the set of all integers
(b) N, the set of all natural numbers
(c) ϕ, the empty set
(d) R, the set of all real numbers

2. Consider a triangle with sides 3 cm, 6 cm, 8 cm respectively. Now if a man runs around the triangle in such a way that he is always at a distance of 1 cm from the side of triangle, then distance travelled by him is given by
(a) 17 (b) 20 (c) $17 + 2\pi$ (d) $17 + 3\pi$

3. The angle between a pair of tangents drawn to the curve $7x^2 - 12y^2 = 84$ from $M(1, 2)$ is given by
(a) $\pi/4$ (b) $\pi/2$ (c) $\pi/6$ (d) $\pi/3$

4. The position vector of a point in which a line through the origin perpendicular to the plane $2x - y - z = 4$ meet the plane $\vec{r} \cdot (3\hat{i} - 5\hat{j} + 2\hat{k}) = 6$, is
(a) $(1, -1, -1)$ (b) $(-1, -1, 2)$
(c) $(4, 2, 2)$ (d) $\left(\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}\right)$

5. Let S be an interval in which $f(x) = \int_0^x \frac{dt}{1+t^3}$ is an invertible function. Suppose g is the inverse of f, then which of the following statement is true?
(a) $g'(x) = \frac{1}{f(f'(x))}$ (b) $g'(x) = \frac{1}{g(g'(x))}$
(c) $g'(x) = \frac{1}{g(f'(x))}$ (d) $g'(x) = \frac{1}{f'(g(x))}$

6. Let f and g are the two functions symmetric about the line $x = a$ and $h(x)$ be a function such that $3h(x) - 4h(2a - x) = 5$. Then the value of $\frac{\int_0^{2a} f(x) \cdot g(x) \cdot h(x)\, dx}{\int_0^{2a} f(x) \cdot g(x)\, dx}$ is
(a) -2 (b) 5 (c) 3 (d) -5

7. Let a relation R in the set N of natural numbers be defined as $(x, y) \in R$ if and only if $x^2 - 4xy + 3y^2 = 0$ for all $x, y \in N$. Then the relation R is
(a) Reflexive (b) Symmetric
(c) Transitive (d) Equivalence

8. If $\int_0^{n\pi+t} |\sin x|\, dx = k - \cos t$ $(0 \le t \le \pi)$, then the value of k is
(a) $2n$ (b) $4n + 1$ (c) $3n + 1$ (d) $2n + 1$

9. If $\alpha = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then $B = A + A^2 + A^3 + A^4$ is
(a) Singular (b) Non-singular
(c) Symmetric (d) $|B| = 1$

10. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ and $f(0) = 1$ and g be a function satisfying $f(x) + g(x) = x^2$. If the value of $\int_0^1 f(x)g(x)\,dx - e + \frac{e^2}{2}$ is k, then the value of $|2k|$ is

(a) 1 (b) 2 (c) 3 (d) 4

11. Two points A and B are taken on the curve $y = \ln x$ whose abscissa are 1 and e respectively. The chord AB is formed. A point on the curve at which the tangent line is parallel to the chord AB is

(a) $(e + 1, \ln(e + 1))$ (b) $(2, \ln 2)$
(c) $(e - 2, \ln(e - 2))$ (d) $(e - 1, \ln(e - 1))$

12. The cubic equation $4x^3 - 33x^2 + 72x + 6 = 0$ has

(a) one positive real root and two complex roots
(b) one negative real root and two complex roots
(c) all three real roots, one negative and two positive
(d) all three real roots, one positive and two negative

13. Let $f(x) = x^4 e^{-x^2}$, $\forall x \in R$. Then $f(x)$ has

(a) Maxima at $x = 0$ (b) Minima at $x = \pm 2$
(c) Minima at $x = \pm 1$ (d) Maxima at $x = \pm\sqrt{2}$

14. Consider a quadratic equation given by $\sum_{r=1}^{n}\left(3x - \frac{1}{2r+1}\right)\left(2x - \frac{1}{2r+5}\right) = 0$. The product of roots of the equation is

(a) $\frac{n+1}{6n(2n+5)}$ (b) $\frac{n+1}{36(2n+5)}$
(c) $\frac{n+1}{36n(2n+5)}$ (d) $\frac{n+1}{72n(2n+5)}$

15. Let $f : R \to (-1, 1)$ defined by $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. Then f is

(a) one-one onto
(b) one-one but not onto
(c) onto but not one-one
(d) Neither one-one nor onto

16. The length of common tangent of hyperbolas $\frac{x^2}{4} - \frac{y^2}{3} = 1$ and $\frac{x^2}{3} - \frac{y^2}{4} + 1 = 0$ is

(a) $\sqrt{\frac{2}{7}}$ (b) $7\sqrt{2}$ (c) 7 (d) $\frac{7}{\sqrt{2}}$

17. $\int_0^2 \left[\tan^{-1} x\right] dx + \int_0^2 \left[\cot^{-1} x\right] dx =$

(where $[\cdot]$ represents the greatest integer function)

(a) $1 - \cot 2$ (b) $1 + \cot 2$
(c) $2(1 + \cot 2)$ (d) $2(1 - \cot 2)$

18. If C_r stands for ${}^nC_r$, then the sum of the series (where n is an even positive integer)

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2 - 2C_1^2 + 3C_2^2 + \ldots.. + (-1)^n(n+1)C_n^2\right]$$

is equal to

(a) 0 (b) $(-1)^{n/2}(n + 1)$
(c) $(-1)^n(n + 2)$ (d) None of these

19. If area of triangle formed by tangents from the point (x_1, y_1) to the parabola $y^2 = 4ax$ and their chord of contact is

(a) $\frac{(y_1^2 - 4ax_1)^{3/2}}{2a^2}$ (b) $\frac{(y_1^2 - 4ax_1)^{2/3}}{a^2}$
(c) $\frac{(y_1^2 - 4ax_1)^{3/2}}{2a}$ (d) none of these

20. A normal to the parabola $y^2 = 4ax$ with slope 'm' touches the rectangular hyperbola $x^2 - y^2 = a^2$, if

(a) $m^6 + 4m^4 - 3m^2 + 1 = 0$
(b) $m^6 - 4m^4 + 3m^2 - 1 = 0$
(c) $m^6 + 4m^4 + 3m^2 + 1 = 0$
(d) $m^6 - 4m^4 - 3m^2 + 1 = 0$

SECTION B (NUMERICAL VALUE TYPE)

21. If $\alpha \neq 1$, $\alpha^6 = 1$ and $\sum_{r=1}^{6} {}^6C_r \cdot \alpha^{r-1} = x$, then the value of $|x| =$ ________.

22. Let $f(x) = x^2 - bx + c$, b is an odd positive integer, $f(x) = 0$ has two prime numbers as roots and $b + c = 35$. Then the modulus of global minimum value of the function $f(x)$ is ________.

23. Maximum value of $\log_5(3x + 4y)$, if $x^2 + y^2 = 25$ is ________.

24. The coefficient of x^6 in the expansion of $\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}\right)^2$ is given by $\frac{31}{k}$, where k is __________.

25. The number of points at which the function $f(x) = (x - |x|)^2 (1 - x + |x|)^2$ is not differentiable in the interval $(-3, 4)$ is ___________.

26. If $f(x) = \sqrt[3]{\frac{9}{\log_2(3 - 2x)} - 1}$, then the value of a which satisfies $f^{-1}(2a - 4) = 1/2$, is _________.

27. If the line $\frac{x-2}{3}=\frac{y-1}{5}=\frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then the value of $\alpha + \beta$ is ________.

28. Number of solutions of the equation $[\cos x] = \{\sin\{x\}\}$ in $[0, 2\pi]$ are __________.

29. The intercept of the common tangent to the curves $y^2 = 8x$ and $xy + 1 = 0$ on the axis of y is ____________.

30. Two man A and B fire at a target. Suppose $P(A) = 1/3$ and $P(B) = 1/5$ are the probabilities of hitting the target by A and B respectively. The probability that the target got hit, is ___________.

SOLUTIONS

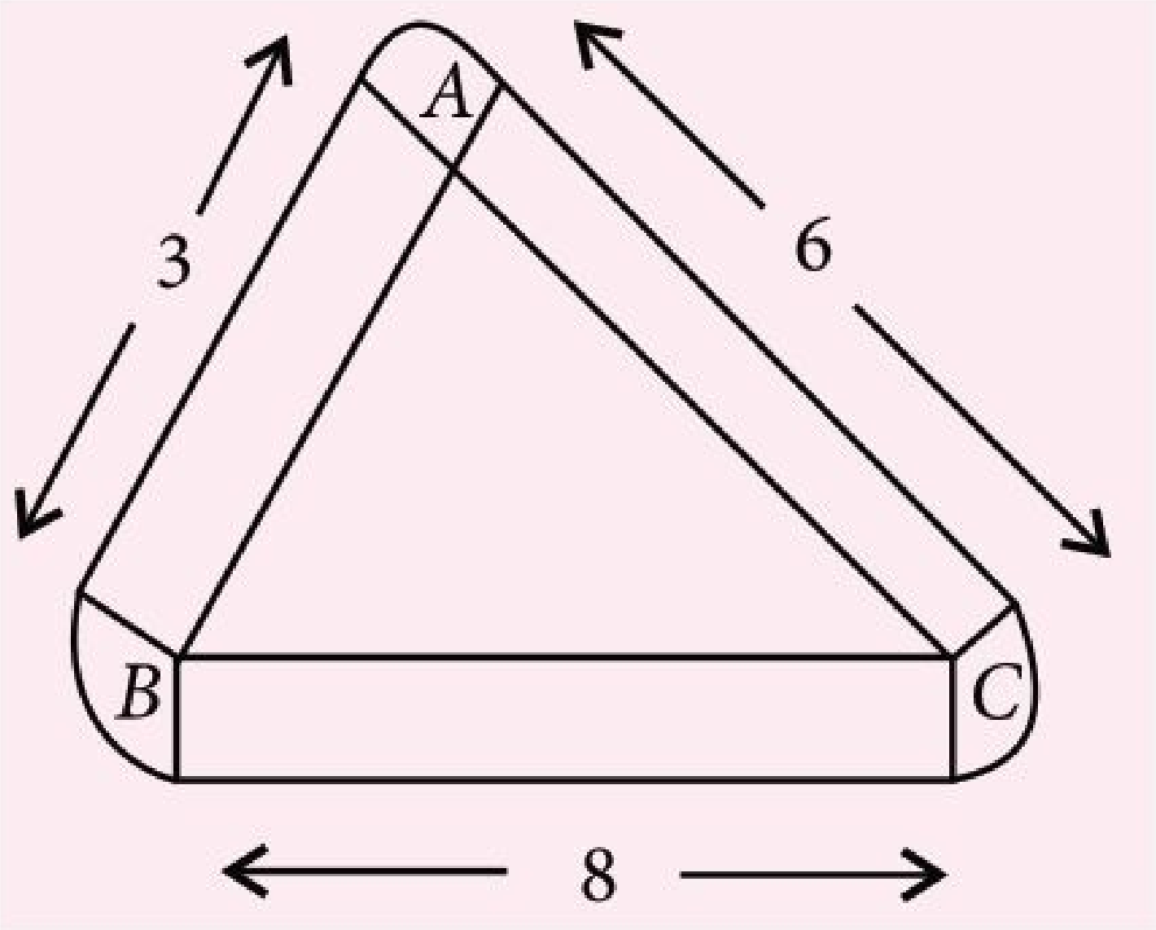
1. (c) : As $f(x) = 1/2$

$\Rightarrow$ $x - [x] = 1$, which is not true

as $x - [x] \in [0, 1)$

i.e., given set is the empty set.

2. (c) :



Distance travelled by man is

$$3 + 6 + 8 + (\pi - A) \times 1 + (\pi - B) \times 1 + (\pi - C) \times 1$$
$$= 17 + 3\pi - (A + B + C) = 17 + 2\pi$$

3. (b) : The equation of pair of tangents from (x_1, y_1) on curve $\equiv SS_1 = T^2$

$$\therefore \left(\frac{x^2}{12}-\frac{y^2}{7}-1\right)\left(\frac{1^2}{12}-\frac{2^2}{7}-1\right)=\left(\frac{1\cdot x}{12}-\frac{y(2)}{7}-1\right)^2$$

$$\Rightarrow \left(\frac{x^2}{12}-\frac{y^2}{7}-1\right)\left(\frac{7-48-84}{84}\right)=\left(\frac{7x-24y-84}{84}\right)^2$$

On solving, we have

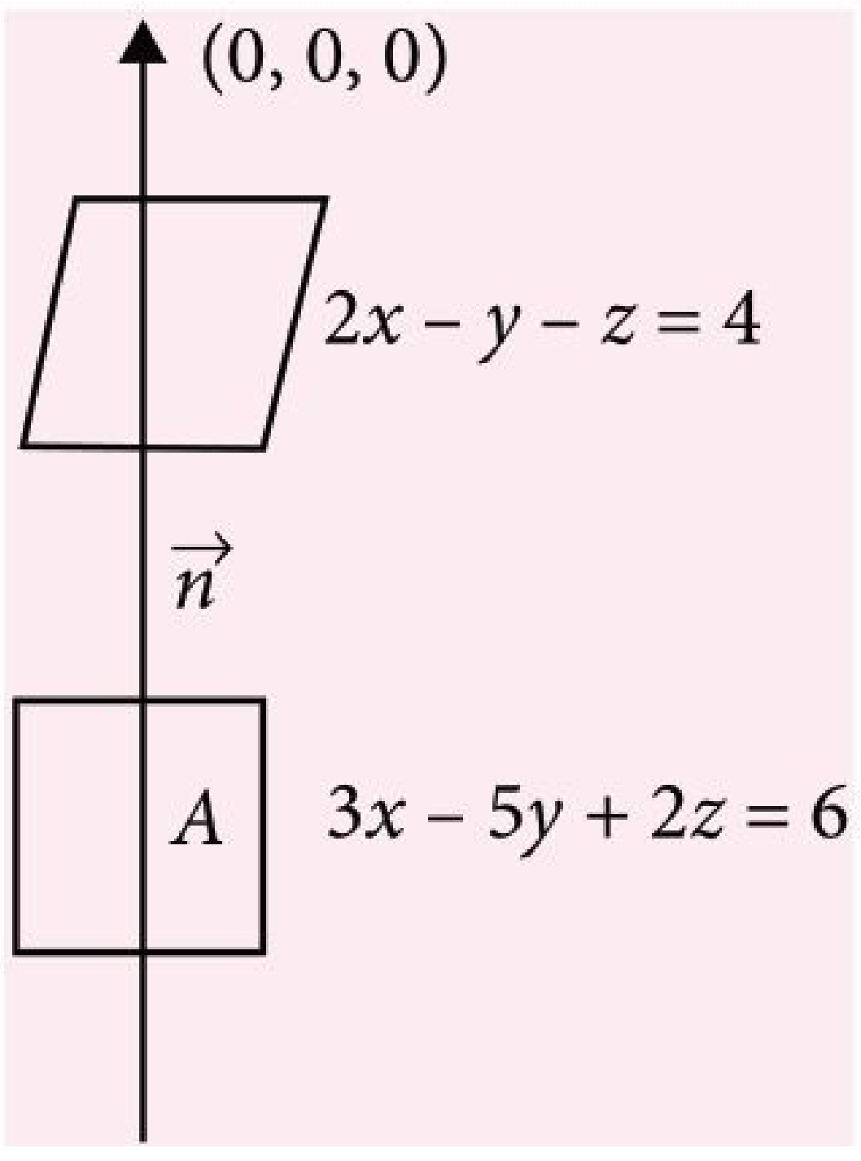
$$(7x^2 - 12y^2 - 84)(7 - 48 - 84) = (7x - 24y - 84)^2$$

$\Rightarrow$ $924x^2 - 924y^2 - 336xy + \alpha x + \beta y + c = 0$

$\because$ Coefficient of x^2 + coefficient of $y^2 = 0$

$\therefore$ Angle subtended is $\pi/2$

4. (d) :



Vector perpendicular to $2x - y - z = 4$ is given by

$$\vec{n} = 2\hat{i} - \hat{j} - \hat{k}$$

Also, line is collinear with $\vec{n}$.

So, equation of line is

$$\vec{r} = \lambda(2\hat{i} - \hat{j} - \hat{k}) \quad \ldots\text{(i)}$$

Now, line (i) meets the plane

$$\vec{r}\cdot(3\hat{i} - 5\hat{j} + 2\hat{k}) = 6$$

So, $\lambda(6 + 5 - 2) = 6 \Rightarrow \lambda = 2/3$

Hence, P.V. of the point A is $\frac{2}{3}(2\hat{i} - \hat{j} - \hat{k})$

i.e. $\left(\frac{4}{3}, \frac{-2}{3}, \frac{-2}{3}\right)$

5. (d) : $\because$ $f'(x) = \frac{1}{1+x^3}$

Also, $f(g(x)) = \int_0^{g(x)} \frac{dt}{1+t^3}$ $\quad (\because\ g = f^{-1})$

$$\Rightarrow x = \int_0^{g(x)} \frac{dt}{1+t^3}$$

Now, $1 = \frac{1}{1+(g(x))^3}\, g'(x)$

$\Rightarrow$ $g'(x) = 1 + (g(x))^3$

$$= \frac{1}{f'(g(x))}$$

6. (d) : $\because$ $f(a - x) = f(a + x)$

or, $f(x + 2a) = f(-x) \Rightarrow f(x) = f(2a - x)$

Similarly, $g(2a - x) = g(x)$

Now, let $I = \int_0^{2a} f(x)\cdot g(x)\cdot h(x)dx$

$$= \int_0^{2a} f(x)\cdot g(x)\cdot h(2a-x)dx$$

or, $4I = \int_0^{2a} f(x)\cdot g(x)\cdot(3h(x)-5)dx$

$$= 3I - 5\int_0^{2a} f(x)\cdot g(x)dx$$

$$\therefore I = -5\int_0^{2a} f(x)\cdot g(x)dx$$

$$\Rightarrow \frac{\int_0^{2a} f(x)\cdot g(x)\cdot h(x)dx}{\int_0^{2a} f(x)\cdot g(x)dx} = -5$$

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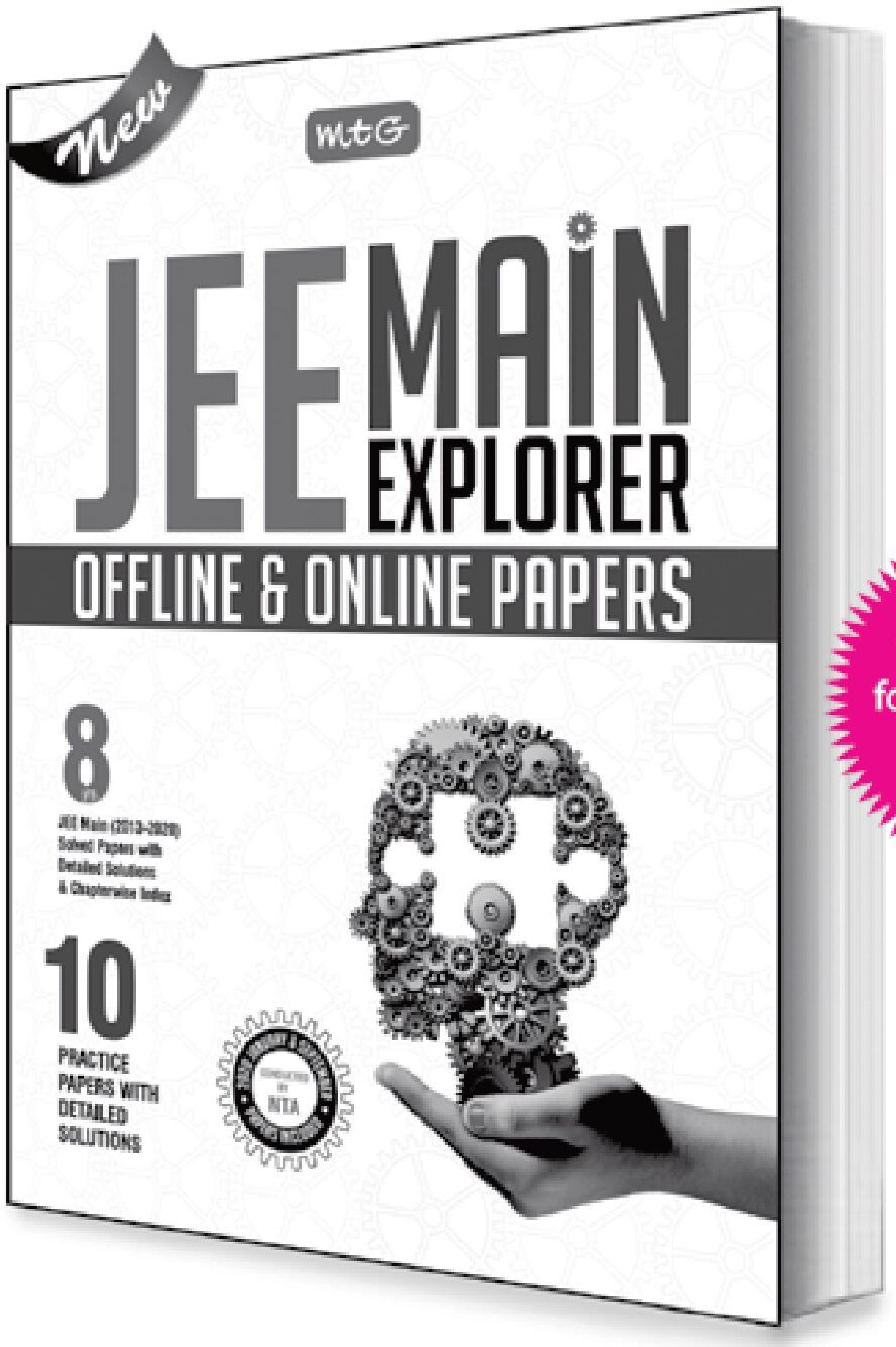
What is JEE (MAIN) like?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7. (a)

8. (d) : $\int_0^{n\pi+t} |\sin x|\,dx = \int_0^{n\pi} |\sin x|\,dx + \int_{n\pi}^{n\pi+t} |\sin x|\,dx$

$= 2n + (-\cos x)_{n\pi}^{n\pi+t}$

$= 2n + \cos(n\pi) - \cos(n\pi + t)$

$= \begin{cases} 2n + 1 - \cos t; & n = \text{even} \\ 2n - 1 + \cos t; & n = \text{odd} \end{cases}$

9. (b) : $\because\ B = A + A^2 + A^3 + A^4$

$= \frac{A(I - A^4)}{I - A} = \frac{A - A^5}{I - A}$

Now, $A^5 = \begin{bmatrix} \cos 5\alpha & \sin 5\alpha \\ -\sin 5\alpha & \cos 5\alpha \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$

10. (c) : $\because\ f'(x) = f(x)$ and $f(0) = 1$

$\Rightarrow\ f(x) = e^x$

$\therefore\ g(x) = x^2 - e^x$

Now, $\int_0^1 f(x)g(x)\,dx = \int_0^1 e^x(x^2 - e^x)\,dx = e - \frac{e^2}{2} - \frac{3}{2}$

$\Rightarrow\ \int_0^1 f(x)g(x)dx - e + \frac{e^2}{2} = \frac{-3}{2}$

$\Rightarrow\ k = -\frac{3}{2}\ \Rightarrow\ |2k| = 3$

11. (d) : Using LMVT,

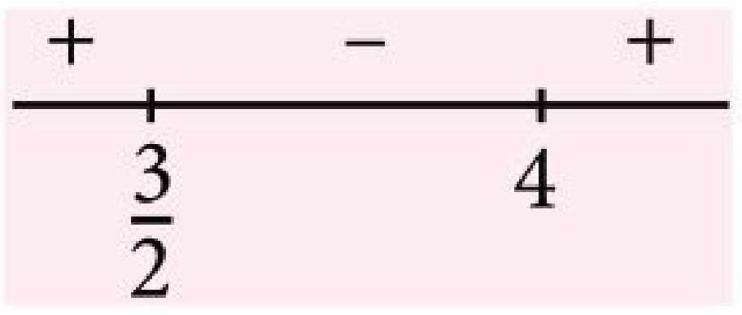
$f'(c) = \frac{f(b) - f(a)}{b - a}\ \Rightarrow\ \frac{1}{c} = \frac{1 - 0}{e - 1}$

$\therefore$ The point on the curve is $(e - 1, \ln(e - 1))$

12. (b) : $f(x) = 4x^3 - 33x^2 + 72x + 6$

$\Rightarrow\ f'(x) = 12x^2 - 66x + 72$

$= 6(2x^2 - 11x + 12) = 6(2x - 3)(x - 4)$



$\Rightarrow\ f(x)$ is increasing in $(-\infty, 3/2) \cup (4, \infty)$ and decreasing in $(3/2, 4)$

$\Rightarrow\ f(4) = 256 - 528 + 288 + 6 > 0$

$\Rightarrow\ f\left(\frac{3}{2}\right) = \frac{27}{2} - \frac{297}{4} + 108 + 6 > 0$

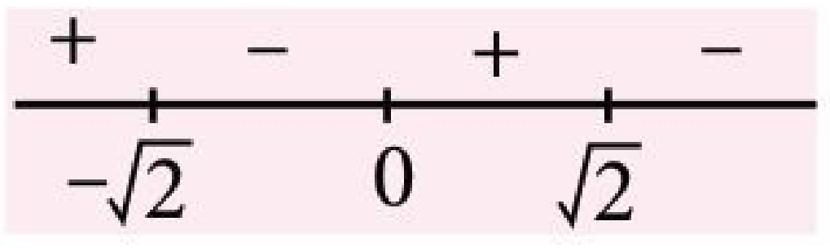
Also, $f(0) = 6 > 0$

$\Rightarrow\ f(x)$ has one negative real root and other two roots are complex.

13. (d) : $f(x) = x^4 e^{-x^2}$

$\Rightarrow\ f'(x) = 4x^3 e^{-x^2} - 2x^5 e^{-x^2}$

$\Rightarrow\ 2x^3 e^{-x^2}(2 - x^2) = 0$



$\Rightarrow\ x = 0, \pm\sqrt{2}$

$\therefore\ f(x)$ has maxima at $x = \pm\sqrt{2}$ and minima at $x = 0$

14. (c) : Let $\frac{1}{2r+1} = \lambda$ and $\frac{1}{2r+5} = \mu$

$\Rightarrow\ \sum_{r=1}^{n} (3x - \lambda)(2x - \mu) = 0$

$\Rightarrow\ \sum_{r=1}^{n} [6x^2 - x(3\mu + 2\lambda) + \lambda\mu] = 0$

Since $\sum_{r=1}^{n} \lambda\mu = \sum_{r=1}^{n} \frac{1}{(2r+1)(2r+5)}$

$\Rightarrow\ \frac{1}{4}\sum_{r=1}^{n}\left(\frac{1}{2r+1} - \frac{1}{2r+5}\right)$

$= \frac{1}{4}\left[\frac{1}{3} - \frac{1}{2n+5}\right] = \frac{n+1}{6(2n+5)}$

$\therefore$ The product of roots of the equation $= \frac{n+1}{36n(2n+5)}$

15. (a) : $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$f'(x) = \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2}$

$= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2}$

$= \frac{2e^x \cdot 2e^{-x}}{(e^x + e^{-x})^2} = \frac{4}{(e^x + e^{-x})^2} > 0, \forall\, x \in R$

$\Rightarrow\ f$ is increasing function for all x in R

$\Rightarrow\ f$ is one-one.

As $x \to \infty$, $f(x) = \frac{1 - e^{-2x}}{1 + e^{-2x}} \to 1$

and $x \to -\infty$, $f(x) = \frac{e^{2x} - 1}{e^{2x} + 1} \to -1$

$\therefore$ Range is $(-1, 1)\ \Rightarrow\ f$ is onto.

16. (b) : $y = mx + \sqrt{4m^2 - 3}$ is a tangent to 1st hyperbola and it becomes a common tangent if $m = \pm 1$. Hence common tangents are $y = \pm x + 1$. The points of contact are $(-4, -3)$ and $(3, 4)$, hence length of common tangent is $7\sqrt{2}$.

17. (c) : $[\tan^{-1}x] = \begin{cases} 0; & 0 \le x < \tan 1 \\ 1; & x \ge \tan 1 \end{cases}$

$[\cot^{-1}x] = \begin{cases} 1; & 0 \le x \le \cot 1 \\ 0; & x > \cot 1 \end{cases}$

$$\int_{\tan 1}^{2} 1\,dx + \int_{0}^{\cot 1} 1\,dx = 2 - \tan 1 + \cot 1 = 2(1 + \cot 2)$$

18. (d) : Let us test the choice for $n = 2$

For $n = 2$, the series

$$= \frac{2(1!)(1!)}{2!}[{}^2C_0^2 - 2\cdot{}^2C_1^2 + 3\cdot{}^2C_2^2] = -4$$

Now observe the table

Choice	Value at $n = 2$
(a)	0
(b)	–3
(c)	4

Since none of the choices become – 4, the correct choice must be (d).

19. (c) : Let $A(x_1, y_1)$ be any point outside the parabola and $B(\alpha, \beta)$, $C(\alpha', \beta')$ be the points of contact of tangents from point A. Eq. of chord BC, $yy_1 = 2a(x + x_1)$

Lengths of ⊥ AL from A to BC is given by

$$AL = \frac{|2a(x_1 + x_1) - y_1y_1|}{\sqrt{y_1^2 + 4a^2}} = \frac{y_1^2 - 4ax_1}{\sqrt{y_1^2 + 4a^2}}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} AL \times BC = \frac{(y_1^2 - 4ax_1)^{3/2}}{2a}$$

20. (c) : Equation of the normal to the parabola $y^2 = 4ax$ with slope m is $y = mx - 2am - am^3$. It touches the rectangular hyperbola $x^2 - y^2 = a^2$ if

$(-2am - am^3)^2 = a^2(m^2 - 1)$

$\Rightarrow a^2m^2(2 + m^2)^2 = a^2(m^2 - 1)$

$\Rightarrow m^2(m^4 + 4m^2 + 4) = m^2 - 1$

$\Rightarrow m^6 + 4m^4 + 3m^2 + 1 = 0$

21. (28) : As, $\alpha^6 = 1$

$$\Rightarrow \alpha = \left(\cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6}\right)^m, \text{ where } m \to 0 \text{ to } 5$$

Taking any one of its roots,

$$\alpha = \cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6}$$

Now, $\sum_{r=1}^{6} {}^6C_r \cdot \alpha^r = \alpha x$

$$\Rightarrow \alpha x = (1 + \alpha)^6 - 1 = \left\{2\cos\frac{\pi}{6}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right\}^6 - 1$$

$$= -27 - 1 = -28$$

$\Rightarrow |\alpha x| = 28 \Rightarrow |x| = 28$

22. (20.25) : Let α, β be the roots of $x^2 - bx + c = 0$, then $\alpha + \beta = b$

$\Rightarrow$ One root is 2, as roots are prime and b is odd integer [∵ α and β both simultaneously can't be odd, so one root must be odd and other even]

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$\therefore\ f(2)=0$

$\Rightarrow\ 2b-c=4$ and $b+c=35$ (given)

So, $b=13$, $c=22$

$$\therefore\ \text{Minimum value} = f\left(\frac{13}{2}\right)=-\frac{81}{4}=-20.25$$

23. (2) : Let $x = 5\cos\theta$, $y = 5\sin\theta$

So, given expression $= \log_5\{5(3\cos\theta + 4\sin\theta)\}$

As $3\cos\theta + 4\sin\theta \le 5$

$\Rightarrow$ Maximum value $= \log_5(25) = 2$

24. (360) : Required coefficient of x^6 is given by

$$2\left\{\frac{1}{1!\times5!}+\frac{1}{2!\times4!}\right\}+\frac{1}{(3!)^2}$$

$$=\frac{1}{6!}\{{}^6C_1+{}^6C_2+{}^6C_3+{}^6C_4+{}^6C_5\}=\frac{2^6-2}{6!}=\frac{31}{360}$$

25. (0) : Given $f(x)$ can be written as

$$f(x)=\begin{cases}16x^4-16x^3+4x^2, & x<0\\ 0, & x\ge 0\end{cases}$$

Clearly, $f(x)$ is continuous as well as derivable $\forall x\in R$.

26. (3) : Given $f^{-1}(2a-4)=\frac{1}{2}$

$$\therefore\ f\left(\frac{1}{2}\right)=2a-4$$

Put $x = 1/2$ in $f(x)=\sqrt[3]{\frac{9}{\log_2(3-2x)}-1}$

We have, $f\left(\frac{1}{2}\right)=\sqrt[3]{9-1}=2$

$\therefore\ 2a-4=2 \Rightarrow a=3$

27. (1) : The required conditions are

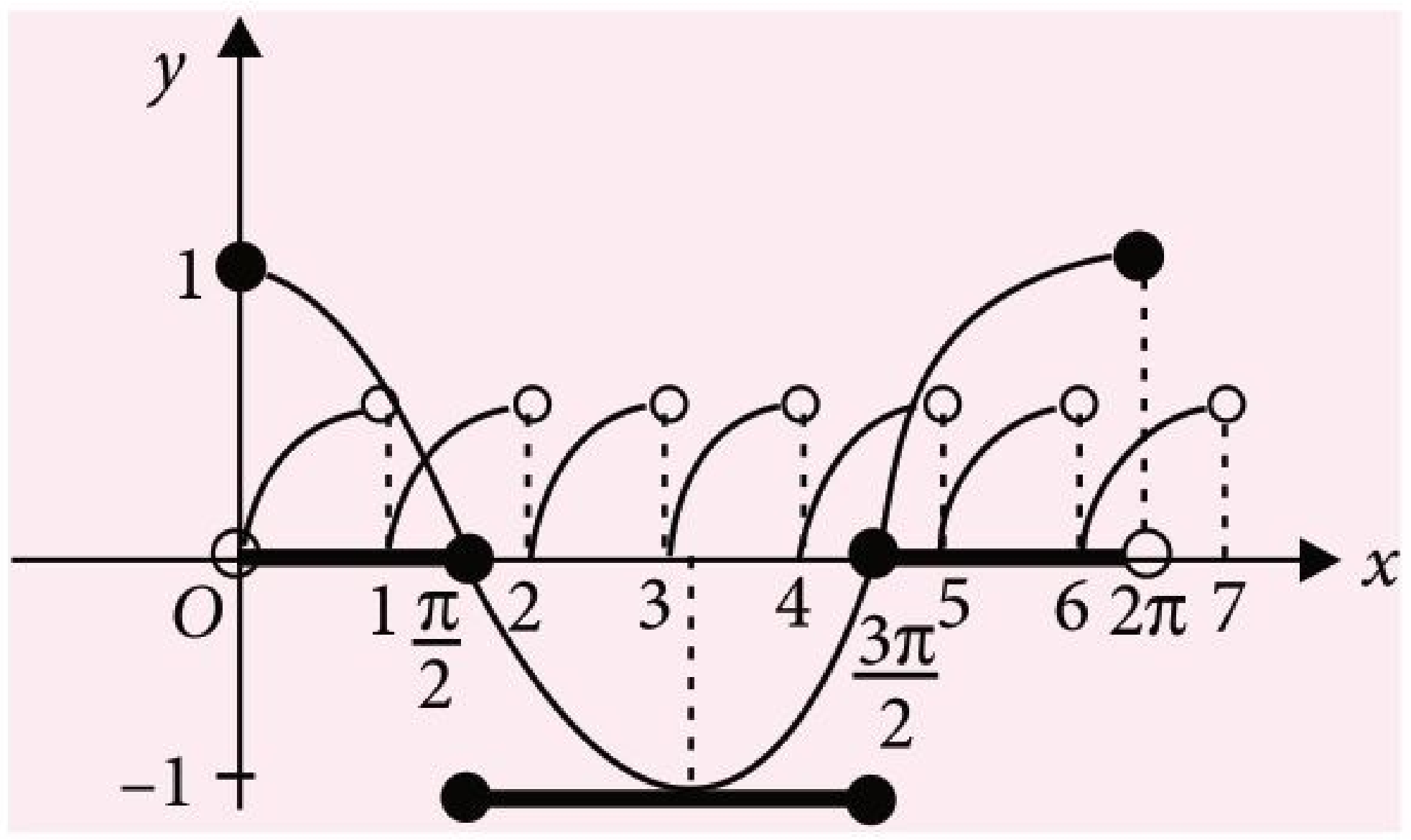
$3-15-2\alpha=0$ ... (i)

and $2+3+2\alpha+\beta=0$ ... (ii)

$\Rightarrow\ \alpha=-6$ and $\beta=7$

$\therefore\ \alpha+\beta=1$

28. (3) : From graph it can be seen that number of solutions is 3.



29. (2) : Equation of tangent of $y^2 = 8x$ is

$$y=mx+\frac{2}{m} \quad \text{... (i)}$$

and $xy+1=0$ ... (ii)

From (i) and (ii)

$$x\left(mx+\frac{2}{m}\right)+1=0 \quad \text{or} \quad mx^2+\frac{2}{m}x+1=0$$

$\Rightarrow\ D=0$

$$\Rightarrow\ \frac{4}{m^2}=4m \Rightarrow m^3=1$$

$\therefore\ m=1$

$\therefore$ Common tangent is $y=x+2$

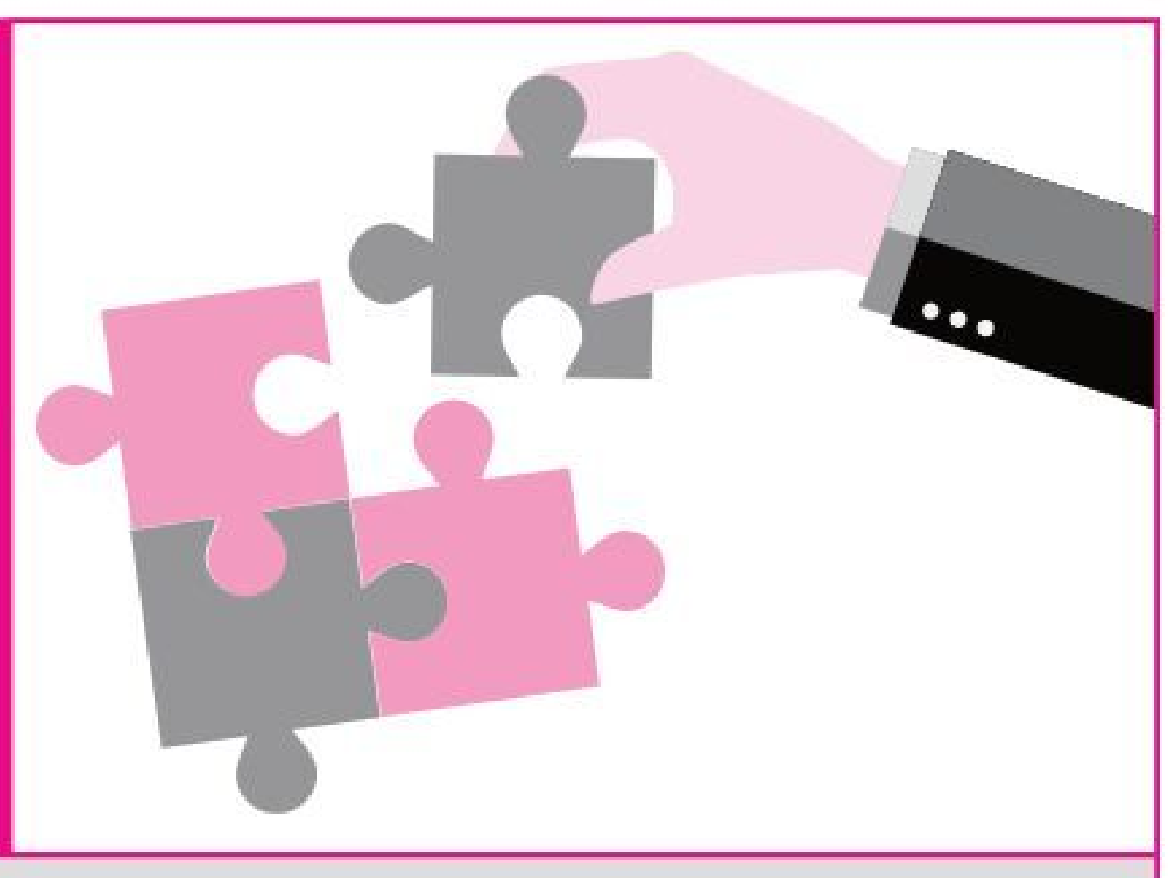
$\therefore$ y-intercept $= 2$

30. (0.46) : $P(E)=P(A)P(B')+P(A')P(B)+P(A)\cdot P(B)$

$$=\left(\frac{1}{3}\times\frac{4}{5}\right)+\left(\frac{2}{3}\times\frac{1}{5}\right)+\left(\frac{1}{3}\times\frac{1}{5}\right)=\frac{4}{15}+\frac{2}{15}+\frac{1}{15}=\frac{7}{15}=0.46$$

◆

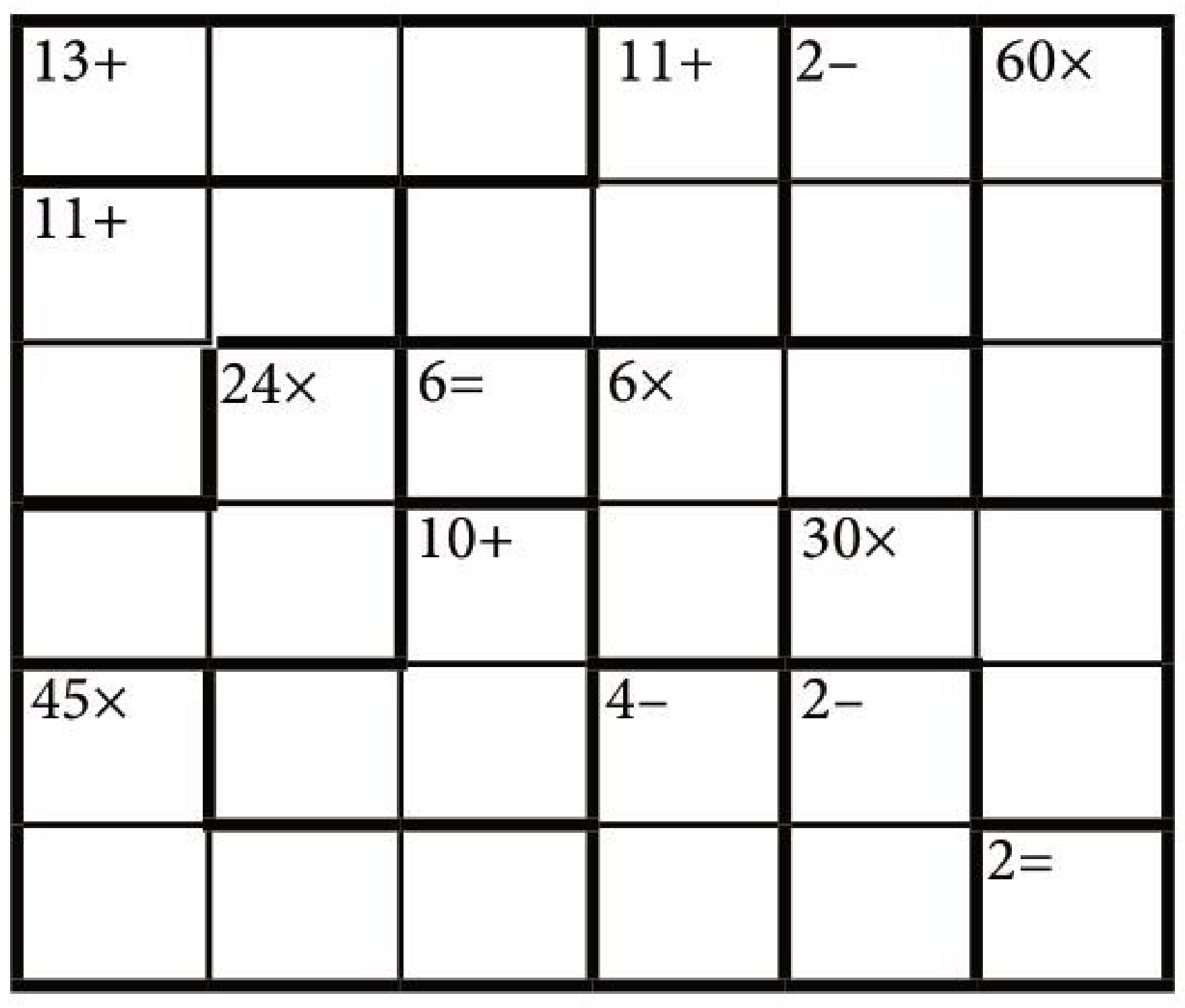
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



13+			11+	2−	60×
11+					
	24×	6=	6×		
		10+		30×	
45×			4−	2−	
					2=

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JEE ADVANCED

PRACTICE QUESTIONS

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SECTION-I

Single Option Correct Type

1. Number of six-digit number in which no digit is repeated, which is divisible by 5, odd numbers occur at even place and even numbers occur at odd place, is

(a) 400 (b) 625 (c) 225 (d) 576

2. The value of $\frac{3+6}{4^{100}}+\frac{3+2\times 6}{4^{99}}+\frac{3+3\times 6}{4^{98}}+\ldots+\frac{3+100\times 6}{4}$ is equal to

(a) $\frac{1}{3}\left(601-\left(\frac{1}{4}\right)^{100}\right)$ (b) $\frac{1}{3}\left(600-\left(\frac{1}{4}\right)^{100}\right)$

(c) $\frac{1}{3}\left(700-\left(\frac{1}{4}\right)^{99}\right)$ (d) $\frac{1}{3}\left(700-\left(\frac{1}{4}\right)^{100}\right)$

3. The family of lines $(2\cos\theta + 3\sin\theta)x + (3\cos\theta - 5\sin\theta)y - (5\cos\theta - 2\sin\theta) = 0$ always passes through the point (α, β) $\forall\ \theta \in R$ then $\alpha + \beta$ is equal to

(a) 1 (b) –1 (c) 2 (d) 0

4. The plane $x - 2y + 3z = 0$ is rotated through a right angle about its line of intersection with the plane $2x + 3y - 4z - 5 = 0$. The equation of plane in its new position is

(a) $x + 3y - 5z = 17$ (b) $11x + 3y - z = 10$

(c) $3x + 4y - 11z = 12$ (d) $22x + 5y - 4z = 35$

5. $\int\frac{3x^2+4x-1}{(x^2+1)^2\sqrt{x+1}}dx$ is equal to

(a) $\frac{\sqrt{x+1}}{x^2+1}+c$ (b) $-\frac{2\sqrt{x+1}}{x^2+1}+c$

(c) $\frac{2}{(x^2+1)\sqrt{x+1}}+c$ (d) $-\frac{x}{(x^2+1)\sqrt{x+1}}+c$

6. The general solution of the differential equation $y\{xy-(x^2-y^2)^2\}dx=\{y^3-x(x^2-y^2)^2\}dy$ is equal to

(a) $\frac{y}{x}+\frac{1}{x^2-y^2}=c$ (b) $\frac{x}{y}-\frac{1}{2}\ln\left|\frac{x-y}{x+y}\right|=c$

(c) $\frac{2x}{y}+\frac{1}{x^2-y^2}=c$ (d) $\frac{2x}{y}-\frac{1}{x^2-y^2}=c$

7. A line is drawn at an angle θ with positive direction of x-axis. It intersects the parabola $y^2 = 8x$ at A and B. If AB is a normal chord which subtend a right angle at the vertex of the parabola, then $\tan^2\theta$ equals

(a) 1/2 (b) 3/4

(c) 1 (d) 2

8. If $f(x)=\frac{3^{x-3}}{3^{1-x}+3^x}$ for all $x \in Q$, then the value of

$$f\left(\frac{1}{109}\right)+f\left(\frac{2}{109}\right)+f\left(\frac{3}{109}\right)+\ldots+f\left(\frac{108}{109}\right)$$ is

(a) 54 (b) 55

(c) 108 (d) 2

9. The value of $\cot\sum_{n=1}^{\infty}\cot^{-1}\left(\frac{2}{n^2}\sum_{k=1}^{n}k^3\right)$ is

(a) 1/2 (b) 1/4

(c) 1/3 (d) 1/6

10. The graph of the function $y = f(x)$ has a unique tangent at $(\pi^3, 0)$. If $A=\lim_{x\to\pi^3}\frac{\ln(1+9f(x))-\sin f(x)}{2f(x)}$, then $\sum_{n=1}^{\infty}A^{-n}$ is equal to

(a) 1/3 (b) 1/2

(c) 4 (d) 1/4

SECTION-II

More than One Options Correct Type

11. If the intercept made by circle $x^2+y^2+x+5y+4=0$ on lines $x+y+1=0$ and $y=m(x+1)-1$ be equal then the values of m can be

(a) 1 (b) -1

(c) $-1/7$ (d) 7

12. If (0, 0) is one of the vertices and $x-y-2=0$ is one of the sides of an equilateral triangle, then

(a) area of triangle is $\frac{2}{\sqrt{3}}$ sq. unit

(b) orthocentre is $\left(\frac{2}{3},-\frac{2}{3}\right)$

(c) slope of other two sides of triangle are $-(2+\sqrt{3})$ and $(\sqrt{3}-2)$

(d) circumcentre is $\left(\frac{1}{3},-\frac{1}{3}\right)$

13. A normal, drawn at a point $P(x, y)$ of a curve meets the x-axis at A and y-axis at B such that $\frac{2}{OA}+\frac{3}{OB}=\frac{1}{5}$, where O is origin. If the curve passes through (7, 11) then

(a) the differential equation of the curve is $(x-10)dx+(y-15)dy=0$

(b) the differential equation of the curve is $(x-10)dx-(y-15)dy=0$

(c) curve passes through (13, 11)

(d) curve passes through (7, 19)

14. Let z_1 and z_2 be the roots of the equations

$$z^2-2z\cos\frac{2\pi}{7}+1=0 \text{ and}$$

$$z^2-2z\cos\frac{3\pi}{7}+1=0 \text{ respectively.}$$

Then which of the following is/are true?

(a) $z_1^2z_2+\frac{1}{z_1^2z_2}=2$ (b) $z_1^2z_2+\frac{1}{z_1^2z_2}=-2$

(c) $\frac{z_1^3}{z_2^2}+\frac{z_2^2}{z_1^3}=2$ (d) $\frac{z_1^3}{z_2^2}+\frac{z_2^2}{z_1^3}=-2\cos\frac{5\pi}{7}$

15. The function $f(x)$ satisfying $(f(x))^2-4f(x)f'(x)+(f'(x))^2=0$ is given by

(a) $ke^{(2+\sqrt{3})x}$ (b) $ke^{(2-\sqrt{3})x}$

(c) $ke^{(2+\sqrt{6})x}$ (d) $ke^{(2-\sqrt{6})x}$

SECTION-III

Single Digit Integer Answer Type

16. The maximum value of the function

$$g(x)=\frac{106}{3x^4+8x^3-18x^2+60},(x>0) \text{ is \_\_\_\_\_\_.}$$

17. Let $p(x)=2+4x+3x^2+5x^3+3x^4+4x^5+2x^6$ for k with $0<k<5$, define $I_k=\int_0^\infty \frac{x^k}{p(x)}dx$, the value of k for which I_k is smallest is _____.

18. The number of points where $f(x)=|2-|2-|x|||$ is not differentiable is _____.

19. If the sum of first n terms of the A.P. 85, 90, 95, ... is equal to the sum of first $3n$ terms of the A.P. 9, 11, 13, ..., then n equals _____.

20. A nine digit number is formed from 1, 2, 3, 4, 5 such that product of all digits is always 1920. The total number of ways is 393(^9P_r), where the value of r is _____.

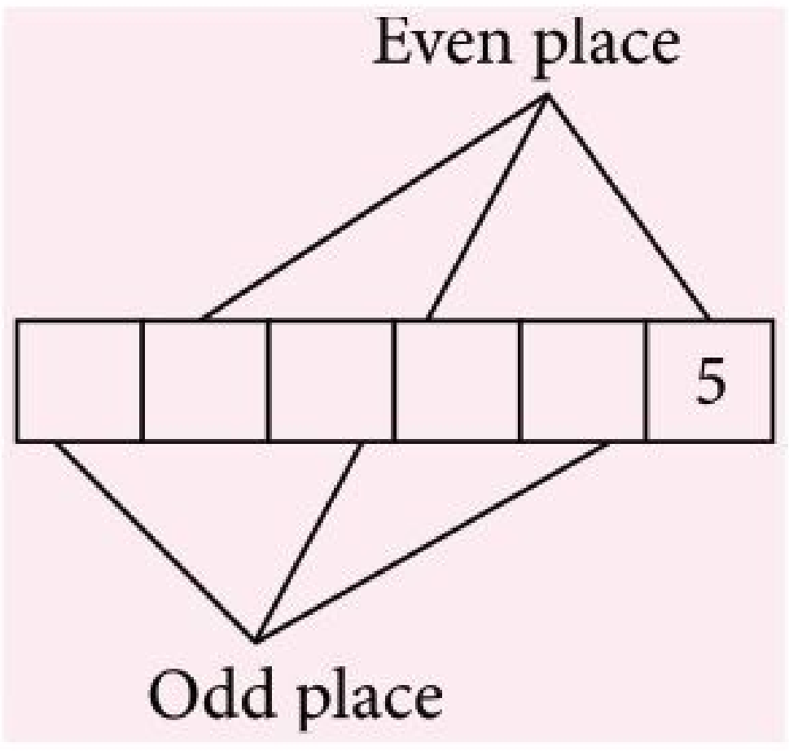
SOLUTIONS

1. **(d) :** Even digits = 0, 2, 4, 6, 8

Odd digits = 1, 3, 5, 7, 9

Required no = 4 × 3 × 4 × 4 × 3

= 576



2. **(a) :**

$$3\underbrace{\left(\frac{1}{4}+\frac{1}{4^2}+....+\frac{1}{4^{100}}\right)}_{S_1},+6\underbrace{\left(\frac{1}{4^{100}}+\frac{2}{4^{99}}+....+\frac{100}{4}\right)}_{S_2}$$

Now, $S_1=\frac{3}{4}\left(\frac{1-\left(\frac{1}{4}\right)^{100}}{1-\frac{1}{4}}\right)=1-\left(\frac{1}{4}\right)^{100}$

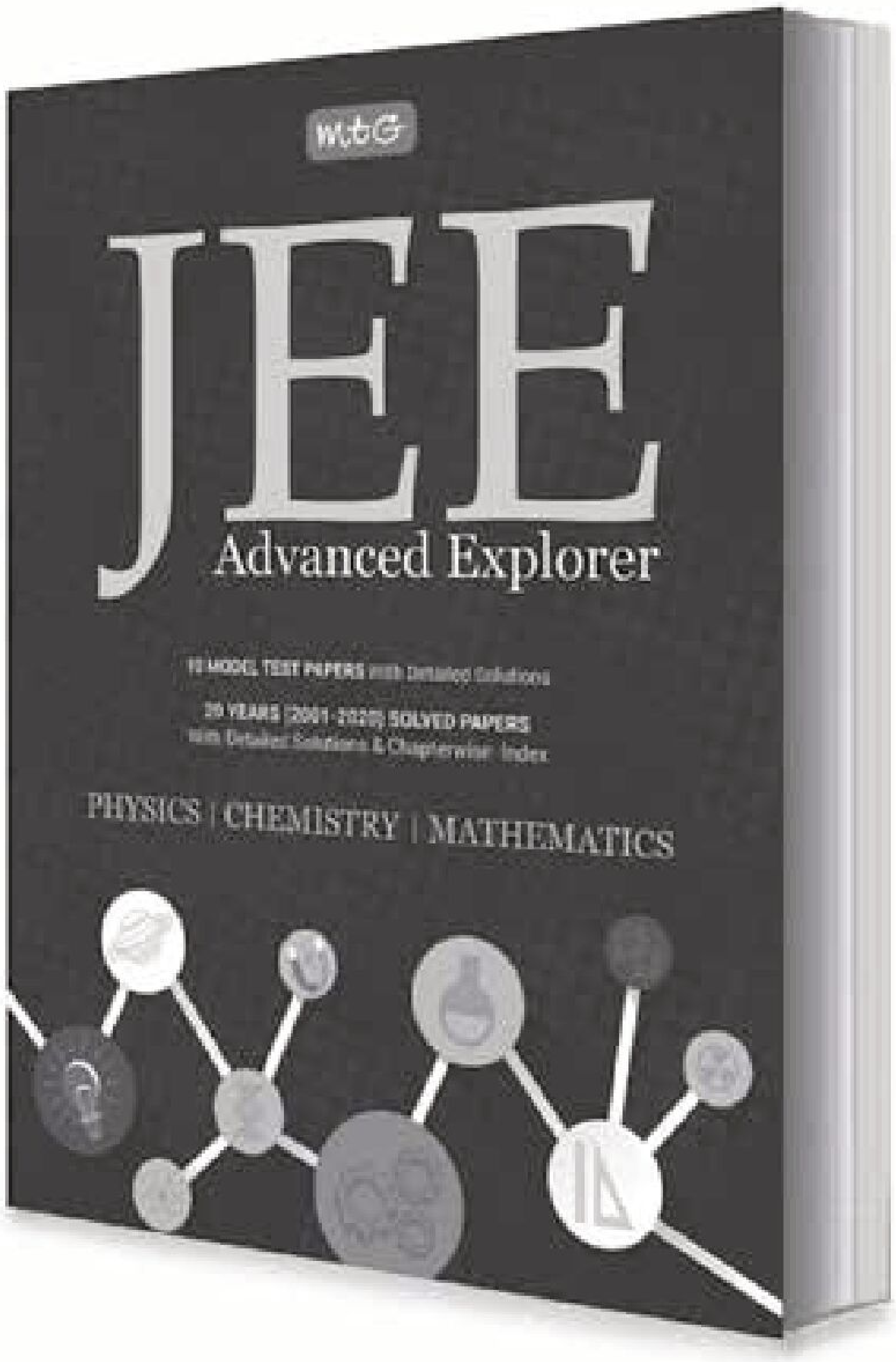
and $S_2=\frac{1}{4^{100}}+\frac{2}{4^{99}}+....+\frac{100}{4}$

or $4S_2=\frac{1}{4^{99}}+.....+100$

$$-3S_2=\frac{1}{4^{100}}+\left(\frac{1}{4^{99}}+\frac{1}{4^{98}}+....+\frac{1}{4}\right)-100$$

$$\Rightarrow\quad S_2=\frac{100}{3}-\frac{1}{9}\left(1-\left(\frac{1}{4}\right)^{100}\right)$$

Precision Revision for JEE is Here!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$$\therefore\ S=\left(1-\left(\frac{1}{4}\right)^{100}\right)+6\left(\frac{100}{3}-\frac{1}{9}\left(1-\left(\frac{1}{4}\right)^{100}\right)\right)$$

$$=\frac{1}{3}\left(601-\left(\frac{1}{4}\right)^{100}\right)$$

3. (c) : $\cos\theta(2x + 3y - 5) + \sin\theta\,(3x - 5y + 2) = 0$
Required point is intersecting point of
$2x + 3y - 5 = 0$ and $3x - 5y + 2 = 0$
$\Rightarrow \alpha = 1, \beta = 1 \quad \therefore \alpha + \beta = 2$

4. (d) : Equation of plane through the line of intersection of $x - 2y + 3z = 0$ and $2x + 3y - 4z - 5 = 0$ is
$x - 2y + 3z + \lambda(2x + 3y - 4z - 5) = 0 \qquad$ ....(i)
(i) is perpendicular to $x - 2y + 3z = 0$
$\therefore\ 1(1 + 2\lambda) - 2(3\lambda - 2) + 3(3 - 4\lambda) = 0$
$\Rightarrow \lambda = 7/8$
$\therefore$ Required equation of plane is $22x + 5y - 4z = 35$

5. (b) : $I=\int\frac{3x^2+4x-1}{(x^2+1)^2\sqrt{x+1}}dx$

$$=\int\frac{4x^2+4x-x^2-1}{(x^2+1)^2\sqrt{x+1}}dx$$

$$=4\int\frac{x(x+1)}{(x^2+1)^2\sqrt{x+1}}dx-\int\frac{dx}{(x^2+1)(\sqrt{x+1})}dx$$

$$=4\int\frac{x(x+1)}{(x^2+1)^2\sqrt{x+1}}dx-$$

$$\left\{\frac{1}{(x^2+1)}2\sqrt{x+1}-\int\frac{-2x}{(x^2+1)^2}2\sqrt{x+1}\,dx\right\}$$

$$=-\frac{2\sqrt{x+1}}{x^2+1}+c$$

6. (c) : $xy^2dx - y(x^2 - y^2)^2dx = y^3dy - x(x^2 - y^2)^2dy$
$\Rightarrow y^2(xdx - ydy) = (x^2 - y^2)^2(ydx - xdy)$

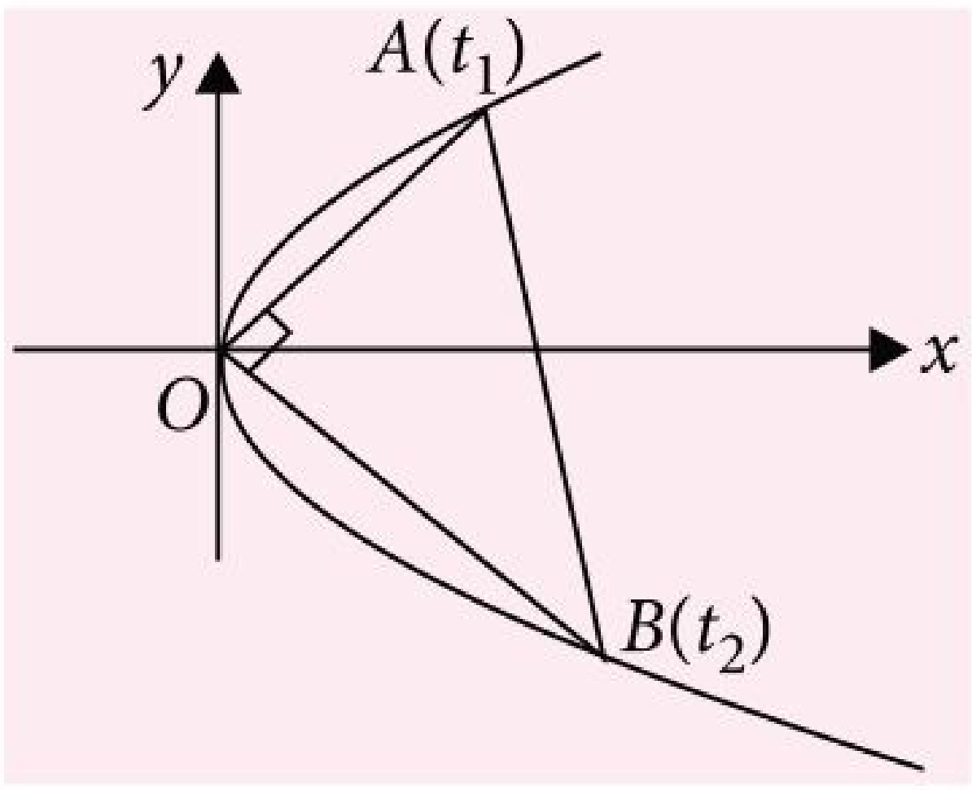
$$\Rightarrow \frac{xdx-ydy}{(x^2-y^2)^2}=\frac{ydx-xdy}{y^2} \Rightarrow \frac{d(x^2-y^2)}{(x^2-y^2)^2}=2d\left(\frac{x}{y}\right)$$

Integrating, we get

$$\frac{2x}{y}+\frac{1}{(x^2-y^2)}=c$$

7. (d) : AB is a normal chord

$$\therefore\ t_2=-t_1-\frac{2}{t_1} \qquad \text{...(i)}$$



Also, AB subtends right angle at the vertex

$$\therefore\ t_1t_2=-4 \Rightarrow t_2=\frac{-4}{t_1} \quad \text{...(ii)}$$

From (i) and (ii), we have

$$-\frac{4}{t_1}=-t_1-\frac{2}{t_1} \ \Rightarrow\ t_1^2=2 \text{ ...(iii)}$$

Equation of AB is $y + t_1x = 2at_1 + at_1^3$
$\therefore$ The slope of $AB = -t_1 = \tan\theta$
$\Rightarrow \tan^2\theta = t_1^2 = 2 \qquad$ (from (iii))

8. (d) : $f(x)+f(1-x)=\frac{1}{27}$

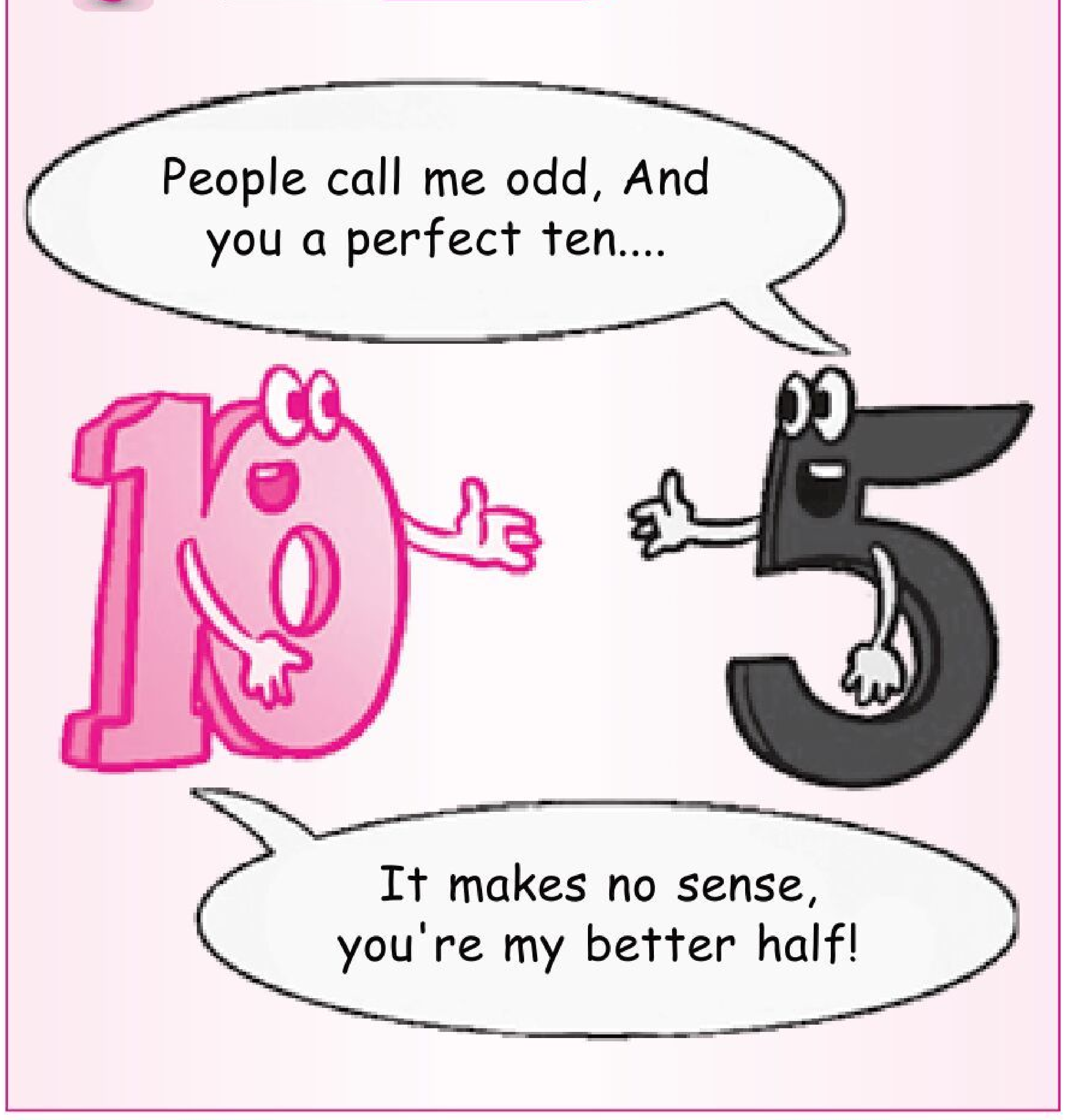
$$\Rightarrow\ \frac{1}{27}\times 54=2$$

9. (c) : $\cot\sum_{n=1}^{\infty}\cot^{-1}\frac{1}{n^2}\times\frac{n^2(n+1)^2}{2}$

$$=\cot\sum_{n=1}^{\infty}\tan^{-1}\frac{2}{(n+1)^2}=\cot\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{n+2-n}{1+n(n+2)}\right)$$

$$=\cot\underbrace{\sum_{n=1}^{\infty}\tan^{-1}(n+2)-\tan^{-1}n}_{S_n}$$

COMIC CAPSULE


People call me odd, And you a perfect ten....
It makes no sense, you're my better half!

$$S_n = \begin{bmatrix} \tan^{-1}3 - \tan^{-1}1 \\ \tan^{-1}4 - \tan^{-1}2 \\ \tan^{-1}5 - \tan^{-1}3 \\ \vdots \\ \tan^{-1}(n+2) - \tan^{-1}(n) \end{bmatrix}$$

$S_n = \tan^{-1}(n + 2) + \tan^{-1}(n + 1) - \tan^{-1} 2 - \tan^{-1}1$

$$S_n = \tan^{-1}\left(\frac{3n^2+7n}{n^2+9n+10}\right),\ S_\infty = \tan^{-1}3$$

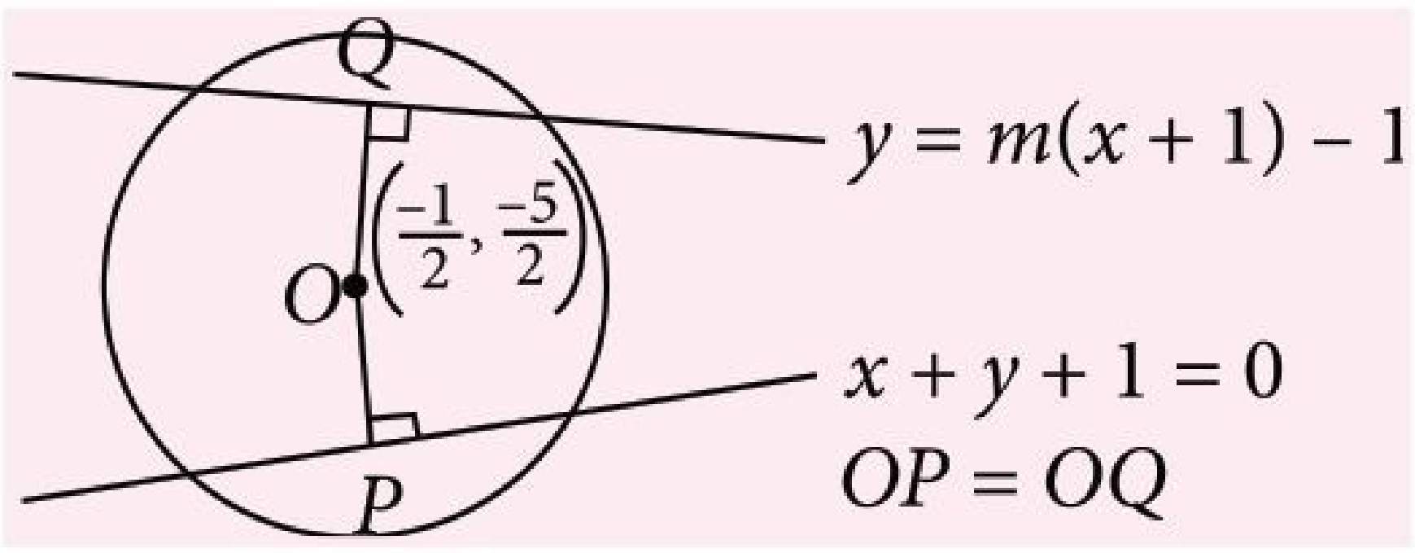
Then $\cot \tan^{-1} 3 = \cot\ \cot^{-1}\left(\frac{1}{3}\right) = \frac{1}{3}$

10 (a) : $\lim\limits_{x\to\pi^3} \dfrac{\left\{\dfrac{1}{1+9f(x)}\right\}9f'(x) - \cos f(x)\cdot f'(x)}{2f'(x)} = \dfrac{9-1}{2} = 4$

$$\sum_{n=1}^{\infty} 4^{-n} = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots\ldots\ldots \infty\ \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{3}$$

11. (a, c) : $\left|\dfrac{-\frac{1}{2}-\frac{5}{2}+1}{\sqrt{2}}\right| = \left|\dfrac{\frac{3}{2}+\frac{m}{2}}{\sqrt{1+m^2}}\right|$

$$\Rightarrow\ 2 = \frac{\frac{9}{4}+\frac{m^2}{4}+\frac{3m}{2}}{(1+m^2)}$$



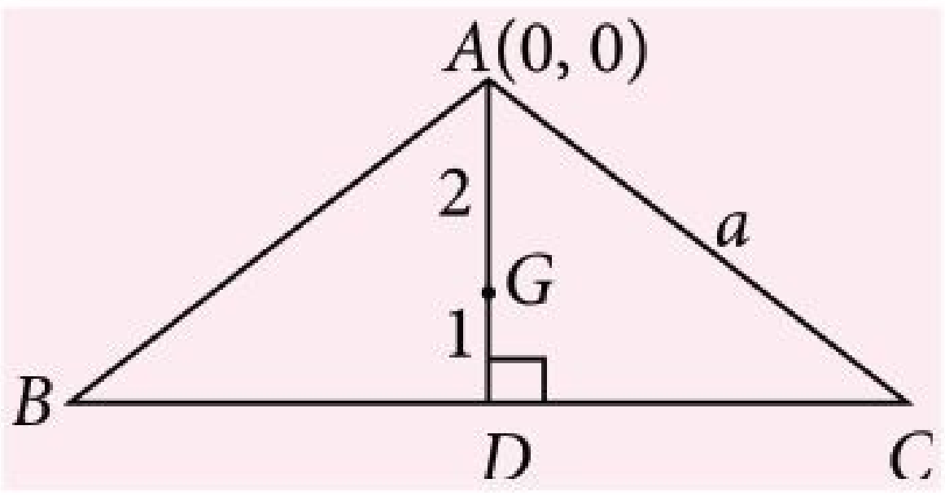
$\Rightarrow\ 2 + 2m^2 = \frac{9}{4} + \frac{m^2}{4} + \frac{3m}{2}\ \Rightarrow\ 7m^2 - 6m - 1 = 0$

$\Rightarrow\ 7m^2 - 7m + m - 1 = 0\ \Rightarrow\ 7m(m - 1) + 1(m - 1) = 0$

$\Rightarrow\ m = 1,\ -\frac{1}{7}$

12. (a, b, c) : Let equation of BC is $x - y - 2 = 0$, Slope of line $x - y - 2 = 0$ is $\tan\theta = 1$

$AD = \left|\frac{2}{\sqrt{2}}\right| = \sqrt{2} = \frac{\sqrt{3}}{2}a$



$\therefore\ a = \frac{2\sqrt{2}}{\sqrt{3}}$

Area $= \frac{\sqrt{3}}{4}\times\frac{8}{3} = \frac{2}{\sqrt{3}}$ sq. unit

Equation of AD is $x + y = 0$

$D = (1, -1)$

$\therefore\ G = \left(\frac{2}{3}, -\frac{2}{3}\right) =$ orthocenter = circumcenter

Slopes of other lines are $\tan(\theta \pm 60) = \frac{1\pm\sqrt{3}}{1\mp\sqrt{3}}$

$= \frac{1+\sqrt{3}}{1-\sqrt{3}},\ \frac{1-\sqrt{3}}{1+\sqrt{3}} = -(2+\sqrt{3}),\ (\sqrt{3}-2)$

13. (a, c, d) : The equation of normal at P is

$$Y - y = -\frac{1}{\left(\frac{dy}{dx}\right)}(X - x)$$

$\Rightarrow\ OA = x + yy'$ and $OB = \frac{x}{y'} + y$

$\therefore\ \frac{2}{OA} + \frac{3}{OB} = \frac{1}{5}$

$\Rightarrow\ \frac{2}{x+yy'} + \frac{3y'}{x+yy'} = \frac{1}{5}$

$\Rightarrow\ 10 + 15y' = x + yy' \Rightarrow (y - 15)dy + (x - 10)dx = 0$

$\Rightarrow\ (x - 10)^2 + (y - 15)^2 = c$, it passes through (7, 11)

$\Rightarrow\ c = 25 \qquad \therefore\ (x - 10)^2 + (y - 15)^2 = 25$

It also passes through (13, 11) and (7, 19).

14. (b, c, d) : $z_1 = \cos\frac{2\pi}{7} \pm i\sin\frac{2\pi}{7} = e^{\frac{2\pi}{7}i}$ or $e^{-\frac{2\pi}{7}i}$

$z_2 = \cos\frac{3\pi}{7} \pm i\sin\frac{3\pi}{7} = e^{\frac{3\pi}{7}i}$ or $e^{\frac{-3\pi}{7}i}$

$$\therefore\ z_1^2 z_2 + \frac{1}{z_1^2 z_2} = e^{\frac{4\pi}{7}i}\cdot e^{\frac{3\pi}{7}i} + \frac{1}{e^{\frac{4\pi}{7}i}\, e^{\frac{3\pi}{7}i}} = -2$$

$$\frac{z_1^3}{z_2^2} + \frac{z_2^2}{z_1^3} = \frac{e^{\frac{6\pi}{7}i}}{e^{\frac{6\pi}{7}i}} + \frac{e^{\frac{6\pi}{7}i}}{e^{\frac{6\pi}{7}i}} = 2$$

(if z_1 and z_2 are $e^{\frac{2\pi}{7}i}$ and $e^{\frac{3\pi}{7}i}$ respectively)

$$\text{Also } \frac{z_1^3}{z_2^2} + \frac{z_2^2}{z_1^3} = \frac{e^{\frac{6\pi}{7}i}}{e^{\frac{-6\pi}{7}i}} + \frac{e^{\frac{-6\pi}{7}i}}{e^{\frac{6\pi}{7}i}}$$

$$= 2\cos\frac{12\pi}{7} = -2\cos\frac{5\pi}{7}$$

15. (a, b) : $\{f(x)\}^2 - 4f(x)\cdot f'(x) + \{f'(x)\}^2 = 0$

$$\Rightarrow\ f'(x)=\frac{4f(x)\pm\sqrt{16(f(x))^2-4(f(x))^2}}{2}$$

$$\therefore\ \frac{f'(x)}{f(x)}=2\pm\sqrt{3}\quad\therefore\ \log_e f(x)=(2\pm\sqrt{3})x+c$$

$$\Rightarrow f(x)=ke^{(2\pm\sqrt{3})x}$$

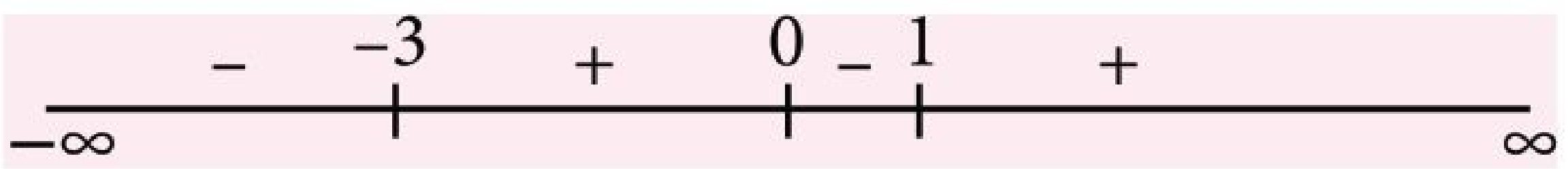
16. (2) : Let $f(x) = 3x^4 + 8x^3 - 18x^2 + 60$

$f'(x) = 12x^3 + 24x^2 - 36x$

$= 12(x^3 + 2x^2 - 3x) = 12x(x^2 + 2x - 3)$

$= 12x(x + 3)(x - 1)$

Also sign of $f'(x)$



$x \neq -3$ and $x = 1$ is point of local minima.

Now, $f(1) = 3 + 8 - 18 + 60 = 53$

$$\therefore\ g(x)_{\max}=\frac{106}{53}=2$$

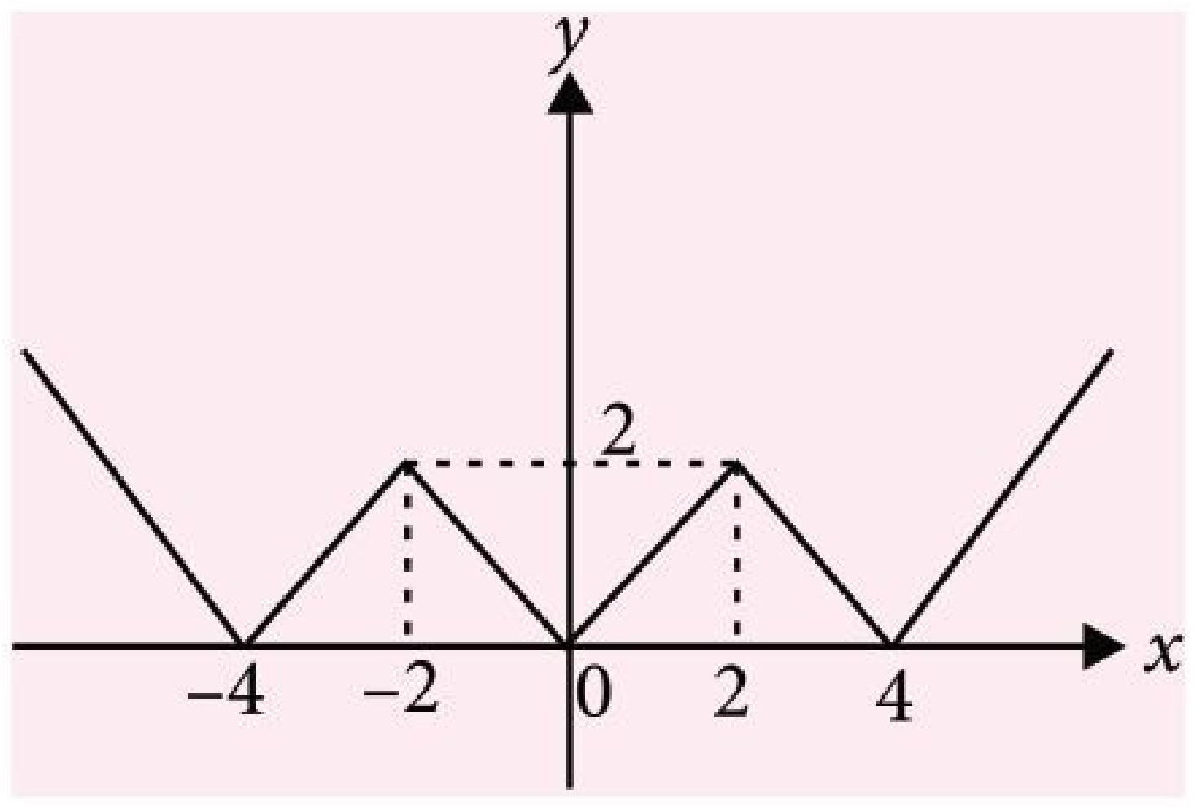
17. (2) : $l_k=\int\limits_{\infty}^{0}\frac{t^{-k}}{t^{-6}p(t)}\left(\frac{-dt}{t^2}\right)=l_{4-k}$

$$l_k=\frac{(l_k+l_{4-k})}{2}=\int\limits_0^{\infty}\frac{(x^k+x^{4-k})}{2p(x)}dx\ \geq\int\limits_0^{\infty}\frac{x^2dx}{p(x)}=l_2$$

$$\left(\frac{x^k+x^{4-k}}{2}\geq x^2\ \text{(AM-GM inequality)}\right)$$

l_k is smallest for $k = 2$.

18. (5) : As shown in the figure, $f(x)$ is non differentiable at 5 points.



19. (9) : $\frac{n}{2}[2\times85+(n-1)\times5]=\frac{3n}{2}[2\times9+(3n-1)\times2]$

$\Rightarrow\ 5n + 165 = 48 + 18n$

$\Rightarrow\ 13n = 117\ \Rightarrow\ n = 9$

20. (2) : $1920 = 5 \times 3 \times 2^7$

$\Rightarrow\ 1920 = 5 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$$=\frac{9!}{7!}$$

Also, $1920 = 5 \times 3 \times 4 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1$

$$=\frac{9!}{5!}$$

Also, $1920 = 5 \times 3 \times 4 \times 4 \times 2 \times 2 \times 2 \times 1 \times 1$

$$=\frac{9!}{2!2!3!}$$

and $1920 = 5 \times 3 \times 4 \times 4 \times 4 \times 2 \times 1 \times 1 \times 1$

$$=\frac{9!}{3!3!}$$

$$\therefore\ \text{Total number of ways}=\frac{9!}{7!}+\frac{9!}{5!}+\frac{9!}{2!2!3!}+\frac{9!}{3!3!}$$

$$=\frac{9!}{7!}[1+42+210+140]=393\cdot{}^9P_2$$

* * *

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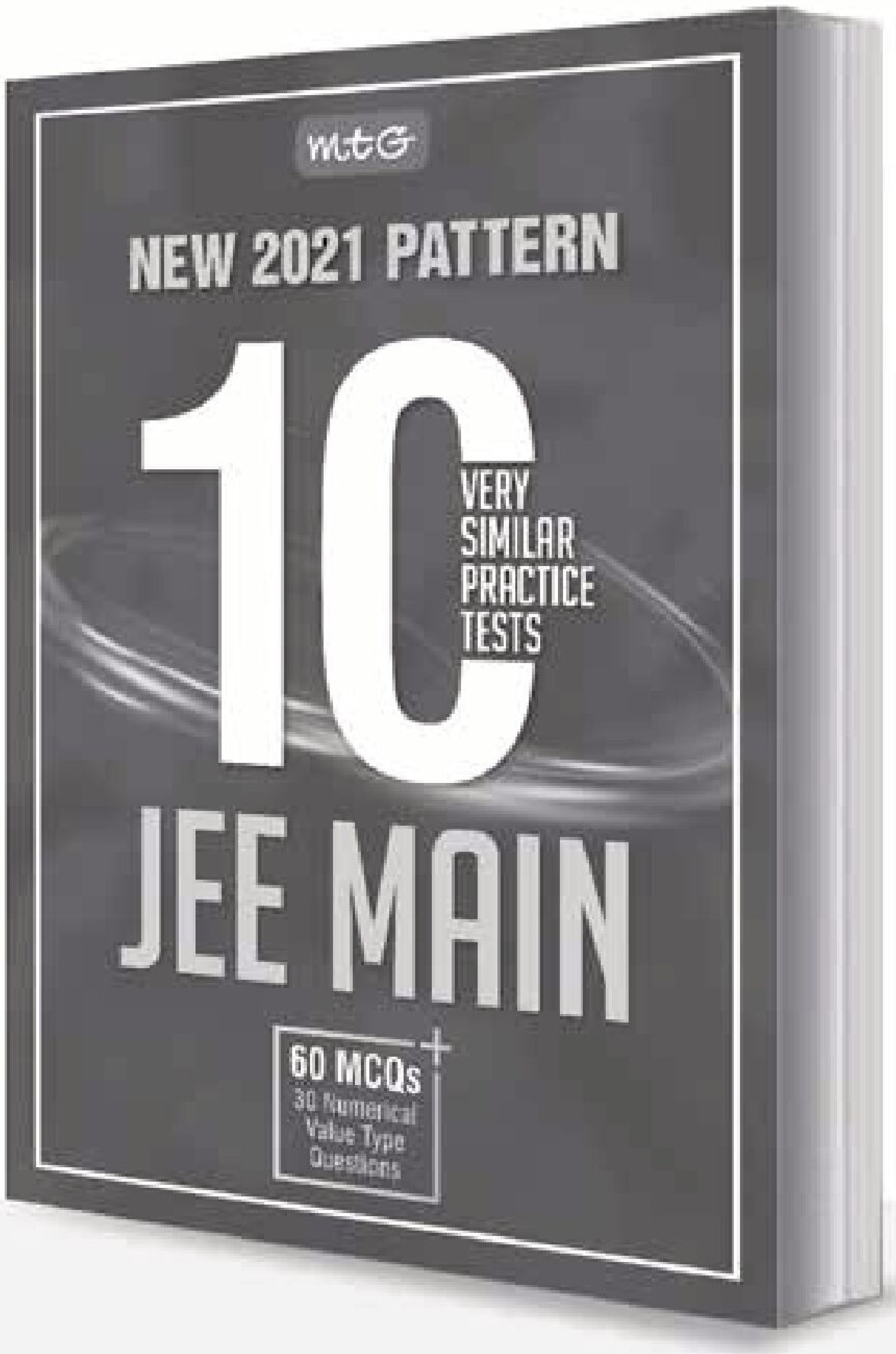

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JEE WORK OUTS

One or More Than One Option(s) Correct Type

1. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to

(a) $\frac{-7\pi}{9}$ (b) $\frac{-2\pi}{9}$ (c) 0 (d) $\frac{5\pi}{9}$

2. Let A and B be two 3×3 matrices of real numbers, where A is symmetric and B is skew-symmetric. If $(A + B)(A - B) = (A - B)(A + B)$ and $(AB)^T = (-1)^k AB$, then the possible value(s) of k is/are

(a) 2 (b) 3 (c) 5 (d) 6

3. If the area enclosed by the parabola $y = 1 + x^2$ and a normal drawn to it with gradient -1, is A then $3A$ is

(a) 2 (b) 3 (c) 4 (d) 5

4. The number of 5 cards combinations out of a deck of 52 cards if there is exactly one ace in each combination is

(a) 768320 (b) 777320 (c) 778230 (d) 778320

5. If $2 \times 2^2 + 3 \times 2^3 + 4 \times 2^4 + \ldots + n \times 2^n = 2^{n+10}$, then the value of n is

(a) 513 (b) 512 (c) 511 (d) 520

6. If $f(x) = \begin{cases} 3 - x^2, & x \leq 2 \\ \sqrt{a+14} - |x - 48|, & x > 2 \end{cases}$ and if $f(x)$ has a local maximum at $x = 2$, then greatest value of a is

(a) 2013 (b) 2012 (c) 2011 (d) 2010

7. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(a) $3a^2 - 26a + 55 = 0$ (b) $3a^2 - 32a + 84 = 0$

(c) $3a^2 - 34a + 91 = 0$ (d) $3a^2 - 23a + 44 = 0$

8. If $y = \sin x - \cos x$ and $f(x) = \frac{d^{17}y}{dx^{17}}$, then the value of $\left(\frac{1024}{\sqrt{2}}\right) f\left(\frac{\pi}{4}\right)$ must be

(a) 1024 (b) 1025

(c) 1022 (d) 1026

9. If $\tan^{-1} y = 4\tan^{-1} x$, then y is infinite if

(a) $x^2 = 3 + 2\sqrt{2}$ (b) $x^2 = 3 - 2\sqrt{2}$

(c) $x^4 = 6x^2 - 1$ (d) $x^4 = 6x^2 + 1$

10. In a ΔABC, $\frac{a}{b} = 2 + \sqrt{3}$ and $\angle C = 60°$. Then the ordered pair $(\angle A, \angle B)$ is equal to

(a) (15°, 105°) (b) (105°, 15°)

(c) (45°, 75°) (d) (75°, 45°)

11. For which values of 'a' will the function $f(x) = x^4 + ax^3 + \frac{3x^2}{2} + 1$ be concave upward along the entire real line ?

(a) $a \in [0, \infty)$ (b) $a \in [-2, \infty)$

(c) $a \in [-2, 2]$ (d) $a \in (0, \infty)$

12. If $\int \frac{dx}{\cos(x-a)\cos(x-b)} = \frac{1}{A}[\log_e(f(x))/\log_e g(x)] + c$, then

(a) $A = \sin(a - b)$ (b) $f(x) = \sec(x - b)$

(c) $g(x) = \sec(x - a)$ (d) $A = \sin(b - a)$

13. Let $f(x) = \cos(\pi(|x| + 2[x]))$, where $[\cdot]$ represents greatest integer function. Then

(a) $f(x)$ is neither odd nor even.

(b) range of $f(x)$ is $[-1, 1]$.

(c) $f(x) = |f(x)|$ for all x.

(d) none of these

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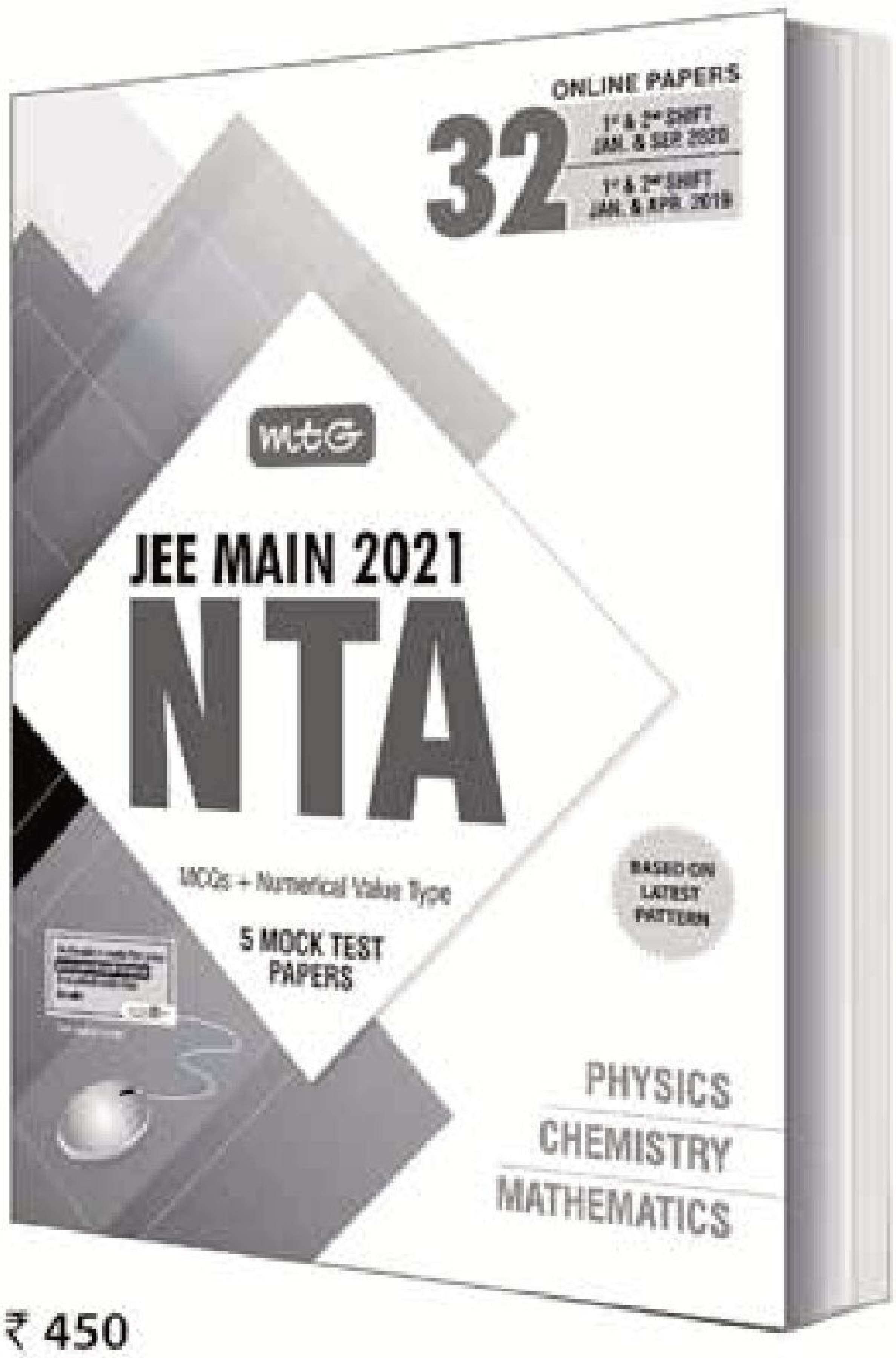
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14. The value of a, for which the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$, $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right angled triangle with $C=\frac{\pi}{2}$ are

(a) 1 (b) 2 (c) – 1 (d) – 2

15. If $f(x)=\frac{e^{2x}-(1+4x)^{1/2}}{\ln(1-x^2)}$ for $x \neq 0$, then f has

(a) an irremovable discontinuity at $x = 0$

(b) a removable discontinuity at $x = 0$ and $f(0) = \frac{1}{4}$

(c) a removable discontinuity at $x = 0$ and $f(0)=-\frac{1}{4}$

(d) none of these

16. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$ if

(a) a, b, c are in A.P.

(b) a, b, c are in G.P.

(c) α is a root of $ax^2 + 2bx + c = 0$

(d) $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$

17. If 2 and 31 appear as two terms in an A.P., then

(a) common difference of the A.P. is a rational number

(b) all the terms of the A.P. must be rational.

(c) all the terms of the A.P. must be integers.

(d) sum to any finite number of terms of the A.P. must be rational.

18. If $f(x) = x^3 - x^2 + 100x + 2002$, then

(a) $f(1000) > f(1001)$ (b) $f\left(\frac{1}{2000}\right) > f\left(\frac{1}{2001}\right)$

(c) $f(x - 1) > f(x - 2)$ (d) none of these

19. Three lines $px + qy + r = 0$, $qx + ry + p = 0$ and $rx + py + q = 0$ are concurrent, if

(a) $p + q + r = 0$

(b) $p^2 + q^2 + r^2 = pq + qr + rp$

(c) $p^3 + q^3 + r^3 = 3pqr$

(d) none of these

20. Consider the points $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$, where $abc \neq 0$; then

(a) the equation of plane ABC is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

(b) the area of ΔABC is $\frac{1}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$

(c) the equation of the plane ABC is $ax + by + cz = 1$

(d) none of these

Comprehension Type

Paragraph for Q. No. 21 to 23

Consider the quadratic polynomial $f(x) = x^2 - 4ax + 5a^2 - 6a$, $(a \in R)$.

21. The value of a for which roots of $f(x) = 0$ are equal in magnitude and opposite in sign, is

(a) 0 (b) 1

(c) 2 (d) none of these

22. Number of values of a for which the equation $f(x) = 0$ has one root equal to zero, is

(a) 0 (b) 1 (c) 2 (d) 3

23. The largest integral value of 'a' for which range of $f(x)$ is $[-5, \infty)$ for every real x, is

(a) 1 (b) 5 (c) 7 (d) 10

Paragraph for Q. No. 24 to 26

Consider the function $f(x)$, a fourth degree polynomial such that $\lim_{x\to -1}\frac{f(x)}{(x+1)^3} = 1$ and $f'''(0) = 0$

24. The maximum value of $f(x)$ is

(a) $\frac{13}{4}$ (b) $\frac{17}{4}$ (c) $\frac{23}{4}$ (d) $\frac{27}{4}$

25. The subnormal of the curve $y = f(x)$ where it cuts the y-axis is

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{1}{2}$

26. The number of inflection points of the curve $y = f(x)$ is

(a) 0 (b) 1 (c) 2 (d) 3

Paragraph for Q. No. 27 to 29

Consider the lines $L_1 : \frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}, L_2 : \frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$

27. The unit vector perpendicular to both L_1 and L_2 is

(a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (d) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

28. The shortest distance between L_1 and L_2 is

(a) 0 (b) $\frac{17}{\sqrt{3}}$ (c) $\frac{41}{5\sqrt{3}}$ (d) $\frac{17}{5\sqrt{3}}$

29. The distance of the point $(1, 1, 1)$ from the plane passing through the point $(-1, -2, -1)$ and whose normal is perpendicular to both L_1 and L_2 is

(a) $\frac{2}{5\sqrt{3}}$ (b) $\frac{7}{5\sqrt{3}}$ (c) $\frac{13}{5\sqrt{3}}$ (d) $\frac{23}{5\sqrt{3}}$

Matrix Match Type

30. Match the following.

	Column-I		Column-II
(A)	If sum of the coefficient of the first, second and third terms in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ is 46, then coefficient of the term that does not contain x is	(p)	3
(B)	For a positive integer n, if the expansion of $\left(\frac{5}{x^2}+x^4\right)^n$ has a term independent of x, then minimum value of n can be	(q)	15
(C)	If in the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$, $n \in N$, sum of the coefficients of x^5 and x^{10} is zero, then the value of n is	(r)	19
(D)	The coefficient of x^4 in the expansion of $(1 + x + x^2)^4$ is	(s)	84

(a) (A) → (p) ; (B) → (r) ; (C) → (q) ; (D) → (s)
(b) (A) → (q) ; (B) → (r) ; (C) → (p) ; (D) → (s)
(c) (A) → (s) ; (B) → (p) ; (C) → (q) ; (D) → (r)
(d) (A) → (r) ; (B) → (q) ; (C) → (p) ; (D) → (s)

31. Consider the circles C_1 of radius a and C_2 of radius b, $b > a$ both lying in the first quadrant and touching the coordinate axes. Find the value of b/a if

	Column-I		Column-II
(A)	C_1 and C_2 touch each other	(p)	$2+\sqrt{2}$
(B)	C_1 and C_2 are orthogonal	(q)	3
(C)	C_1 and C_2 intersect so that the common chord is longest	(r)	$2+\sqrt{3}$
(D)	C_2 passes through the centre of C_1	(s)	$3+2\sqrt{2}$

(a) (A) → (s) ; (B) → (p,s) ; (C) → (q) ; (D) → (q)
(b) (A) → (r) ; (B) → (q) ; (C) → (p,s) ; (D) → (q)
(c) (A) → (p,q) ; (B) → (r) ; (C) → (s) ; (D) → (p)
(d) (A) → (s) ; (B) → (r) ; (C) → (q) ; (D) → (p)

32. Match the following.

	Column-I		Column-II
(A)	$\lim\limits_{x\to 2^+} \{x\}\frac{\sin(x-2)}{(x-2)^2} =$	(p)	– 3
(B)	$\lim\limits_{x\to 2}$ [2 – x] + [x – 2] – x =	(q)	1
(C)	$\lim\limits_{x\to 2}$ {2 – x} + {x – 2} =	(r)	– 1
(D)	$\lim\limits_{x\to 0^+} \frac{e^{\frac{1}{x}}-e^{\frac{-1}{x}}}{e^{\frac{1}{x}}+e^{\frac{-1}{x}}} =$	(s)	2

(a) (A) → (q) ; (B) → (p) ; (C) → (q); (D) → (q)
(b) (A) → (p,r) ; (B) → (r) ; (C) → (p); (D) → (q)
(c) (A) → (p) ; (B) → (q) ; (C) → (r,s); (D) → (p)
(d) (A) → (q) ; (B) → (p,q) ; (C) → (r); (D) → (s)

Numerical Answer Type

33. Let $f: R^+ \to R$ be a function which satisfies $f(x)\cdot f(y) = f(xy) + 2\left(\frac{1}{x}+\frac{1}{y}+1\right)$ for x, y > 0. Then possible integral value of $f(1/2)$ is ______.

34. Water is dropped at the rate of 2 m^3/s into a cone of semi vertical angle 45°. If the rate at which periphery of water surface changes when the height of the water in the cone is 2 m is d, then the value of 5 d is ______.

35. If $\sin\left\{2\left(\sin^{-1}\frac{\sqrt{5}}{3}-\cos^{-1}\frac{\sqrt{5}}{3}\right)\right\}$ is equal to $\frac{k\sqrt{5}}{81}$, then k = ______.

36. If $g(x)$ is a differentiable function $g(1) \neq 0$, $g(-1) \neq 0$ and Rolle's theorem is not applicable to $f(x) = \frac{x^2-1}{g(x)}$ in [–1, 1], then $g(x)$ has atleast k root in (–1, 1), then k = ______.

37. The greatest possible number of points of intersection of 8 straight lines and 4 circles is ______.

38. If $\int (x^9+x^6+x^3)(2x^6+3x^3+6)^{1/3}dx = \frac{1}{A}(2x^9+3x^6+6x^3)^B + K$, then the value of $\frac{AB}{4}$ is ______.

39. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then 16 $P(X \geq 1)$ = ______.

40. Let $\vec{a} = -\hat{i}-\hat{k}, \vec{b} = -\hat{i}+\hat{j}$ and $\vec{c} = \hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{r}\cdot\vec{a} = 0$, then $\vec{r}\cdot\vec{b}$ = ______.

SOLUTIONS

1. **(c) :** $\sqrt{3}\sin x + \cos x = 2\cos 2x$

$$\Rightarrow \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \cos 2x \Rightarrow \cos\left(x-\frac{\pi}{3}\right) = \cos 2x$$

$$\Rightarrow 2x \pm \left(x-\frac{\pi}{3}\right) = 2k\pi, k \in Z$$

Then $x=(6k-1)\frac{\pi}{3}$ or $(6k+1)\frac{\pi}{9}, k\in Z$

The values of x in $(-\pi, \pi)$ are $-\frac{\pi}{3}, \frac{\pi}{9}, \frac{7\pi}{9}, -\frac{5\pi}{9}$

$\therefore$ Required sum = 0

2. **(b, c) :** Since A is symmetric, $A^T = A$.
Since B is skew-symmetric, $B^T = -B$.
Given, $(A + B)(A - B) = (A - B)(A + B)$
$\Rightarrow A^2 - AB + BA - B^2 = A^2 + AB - BA - B^2$
$\Rightarrow 2BA = 2AB \Rightarrow AB = BA$
Also, $(AB)^T = (-1)^k AB$
$\Rightarrow B^TA^T = (-1)^k AB \Rightarrow -BA = (-1)^k AB$
$\Rightarrow -AB = (-1)^k AB \Rightarrow AB = (-1)^{k+1} AB$
$\therefore$ k = odd number

3. **(c) :** Given, $y = 1 + x^2$ ...(i)

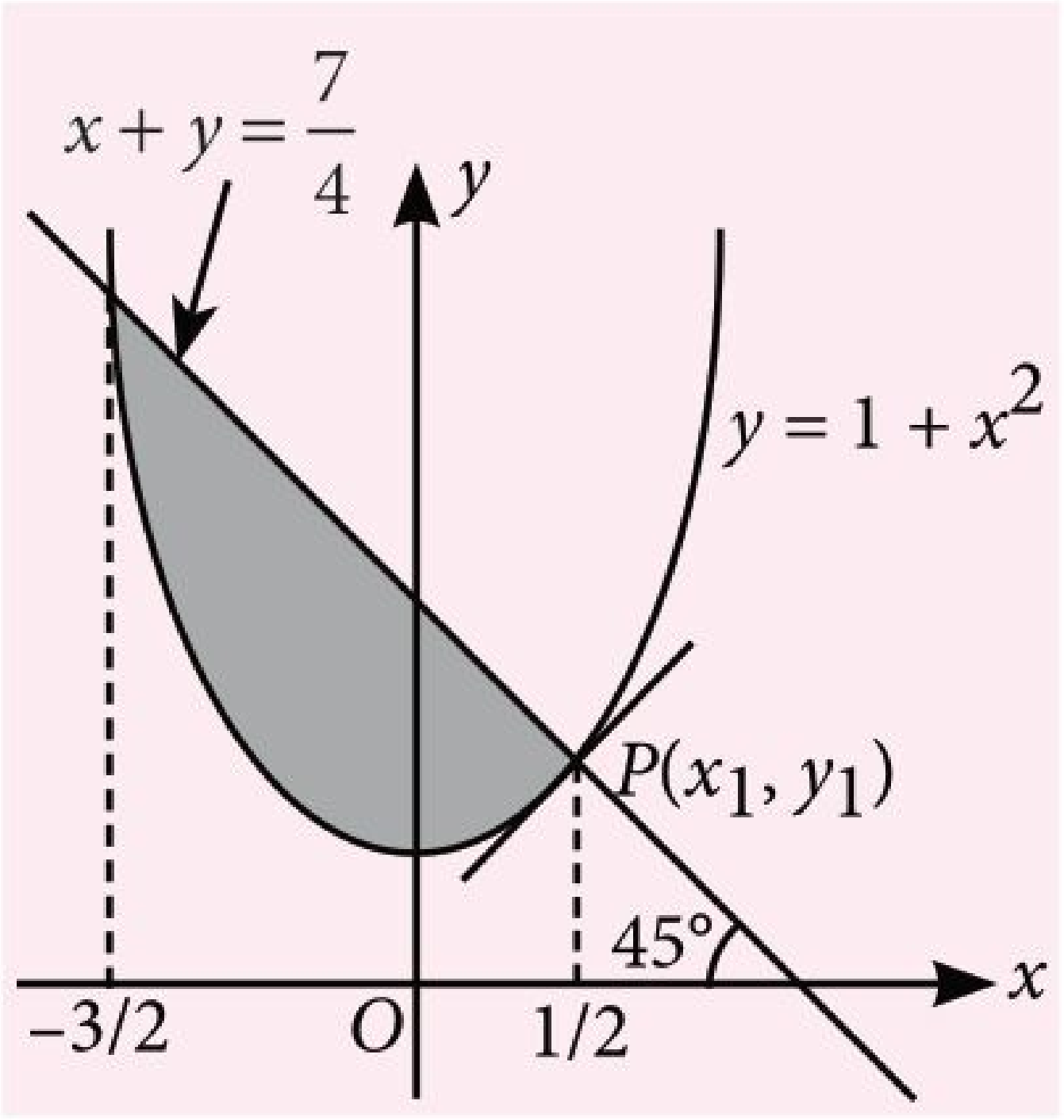
$\Rightarrow \frac{dy}{dx}=2x$

Hence $m=-\frac{1}{2x_1}$ (m = slope of normal at (x_1, y_1))

$\therefore x_1=\frac{-1}{2m}=\frac{1}{2}$; $y_1=1+\frac{1}{4m^2}=\frac{5}{4}$ $[\because m = -1]$

Equation of normal is $y-\frac{5}{4}=-\left(x-\frac{1}{2}\right)$

$\Rightarrow x+y=\frac{5}{4}+\frac{1}{2}=\frac{7}{4}$ ...(ii)



$$A=\int_{x_1}^{x_2}\left(\left(\frac{7}{4}-x\right)-(1+x^2)\right)dx$$

$$=\int_{x_1}^{x_2}\left(\frac{3}{4}-x-x^2\right)dx$$

Now, $1+x^2=-x+\frac{7}{4}$

$\Rightarrow 4x^2+4x-3=0 \Rightarrow x=-\frac{3}{2}$ and $x=\frac{1}{2}$

$$\therefore A=\int_{-3/2}^{1/2}\left(\frac{3}{4}-x-x^2\right)dx=\left[\frac{3}{4}x-\frac{x^2}{2}-\frac{x^3}{3}\right]_{-3/2}^{1/2}$$

$\Rightarrow A=\frac{4}{3}$ sq. units $\Rightarrow 3A = 4$.

4. **(d) :** Total number of cards = 52
Number of ace cards = 4
Number of non-ace cards = 48
$\therefore$ One ace card out of 4 can be selected in ${}^4C_1$ ways.
Remaining 4 cards out of 48 cards can be selected in ${}^{48}C_4$ ways.
$\therefore$ Required number of ways = ${}^4C_1 \times {}^{48}C_4$

$$=\frac{4!}{3!\times 1!}\times\frac{48!}{4!\times 44!}=\frac{48\times 47\times 46\times 45}{6}=778320$$

5. **(a) :** We have, $2^{n+10} = 2 \times 2^2 + 3 \times 2^3 + 4 \times 2^4 + \ldots + (n-1) \times 2^{n-1} + n \times 2^n$ ...(i)

On multiplying both sides by 2, we get
$2 \times 2^{n+10} = 2 \times 2^3 + 3 \times 2^4 + \ldots + (n-1)2^n + n \times 2^{n+1}$ ...(ii)

Subtracting (ii) from (i), we get
$-2^{n+10} = 2 \times 2^2 + (2^3 + 2^4 + \ldots + 2^n) - n \times 2^{n+1}$
$\Rightarrow -2^{n+10} = 8 + 8(2^{n-2} - 1) - n \times 2^{n+1}$
$\Rightarrow -2^{n+10} = 2^{n+1} - n \times 2^{n+1}$
$\Rightarrow -2^{10} = 2 - 2n \Rightarrow n = 513$

6. **(c) :** Here, $f(x)=\begin{cases}3-x^2, & x\le 2\\ \sqrt{a+14}-|x-48|, & x>2\end{cases}$

Since, $f(x)$ has a local maximum at $x = 2$.
$\therefore \lim_{h\to 0} f(2+h)\le f(2)$
$\Rightarrow \lim_{h\to 0}(\sqrt{a+14}-|2+h-48|)\le 3-2^2$
$\Rightarrow \sqrt{a+14}\le 45 \Rightarrow a\le 2011$

7. **(b) :** Since, $\text{S.D.}=\sqrt{\frac{\sum x_i^2}{n}-\left(\frac{\sum x_i}{n}\right)^2}$

Now, $\frac{49}{4}=\frac{4+9+a^2+121}{4}-\left(\frac{16+a}{4}\right)^2$
$\Rightarrow 49 \cdot 4 = 4(134 + a^2) - (256 + 32a + a^2)$
$\Rightarrow 3a^2 - 32a + 84 = 0$

8. **(a) :** Differentiation of sin$x$ – cos$x$ repeat after fourth order.

$\therefore \frac{d^{16}y}{dx^{16}}=\sin x-\cos x \Rightarrow f(x)=\frac{d^{17}y}{dx^{17}}=\cos x+\sin x$

$\Rightarrow f\left(\frac{\pi}{4}\right)=\cos\frac{\pi}{4}+\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\sqrt{2}$

$\therefore \left(\frac{1024}{\sqrt{2}}\right)f\left(\frac{\pi}{4}\right)=\left(\frac{1024}{\sqrt{2}}\right)(\sqrt{2})=1024$

9. **(a, b, c) :** If we put $x = \tan\theta$, the given equality becomes $\tan^{-1} y = 4\theta$.

$$\Rightarrow y=\tan 4\theta=\frac{2\tan 2\theta}{1-\tan^2 2\theta}=\frac{2\left[\frac{2\tan\theta}{1-\tan^2\theta}\right]}{1-\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2}$$

$$=\frac{2\times 2x(1-x^2)}{(1-x^2)^2-4x^2}=\frac{4x(1-x^2)}{1-6x^2+x^4}$$

Now, y is infinite if $x^4 - 6x^2 + 1 = 0$

$\Rightarrow x^2=\frac{6\pm\sqrt{36-4}}{2}=3\pm 2\sqrt{2}$

10. (b) : Since, $\frac{a}{b}=\frac{2+\sqrt{3}}{1} \Rightarrow \angle A > \angle B$

Also, $\frac{a}{b}=\frac{\sin 105^\circ}{\sin 15^\circ}=2+\sqrt{3}$, which is true.

11. (c) : Here, $f(x) = x^4 + ax^3 + \frac{3x^2}{2} + 1$

$\Rightarrow f'(x) = 4x^3 + 3ax^2 + 3x \Rightarrow f''(x) = 12x^2 + 6ax + 3$

Now, $f(x)$ will be concave upward along the entire real line if $f''(x) \geq 0\ \forall\ x \in R$

$\Rightarrow\quad 12x^2 + 6ax + 3 \geq 0\ \forall\ x \in R \quad\Rightarrow\quad D \leq 0$

$\Rightarrow\ 36a^2 - 144 \leq 0 \quad\Rightarrow\quad a^2 - 4 \leq 0 \quad\Rightarrow\quad a \in [-2, 2]$

12. (a, b, c) : $I=\frac{1}{\sin(a-b)}\int\frac{\sin(x-b-(x-a))dx}{\cos(x-a)\cos(x-b)}$

$=\frac{1}{\sin(a-b)}\int(\tan(x-b)-\tan(x-a))dx$

13. (b) : $f(x) = \cos(2\pi[x] + \pi|x|) = \cos(\pi|x|) = \cos(\pi x)$

Clearly $f(x)$ is an even function with range $[-1,1]$.

14. (a, b) : $\overrightarrow{BC} = (a\hat{i} - 3\hat{j} + \hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k})$

$= (a - 1)\hat{i} + 6\hat{k}$

$\overrightarrow{CA} = (2\hat{i} - \hat{j} + \hat{k}) - (a\hat{i} - 3\hat{j} + \hat{k}) = (2 - a)\hat{i} + 2\hat{j}$

$\overrightarrow{BC}\cdot\overrightarrow{CA} = 0 \Rightarrow (a-1)(2-a) = 0$

$\therefore\quad a = 1, 2.$

15. (d) : $\lim\limits_{x\to 0}\frac{e^{2x}-(1+4x)^{1/2}}{\log(1-x^2)}=\lim\limits_{x\to 0}\frac{e^{2x}-(1+4x)^{1/2}}{\frac{\log(1-x^2)}{-x^2}(-x^2)}$

$=\lim\limits_{x\to 0}\frac{(1+4x)^{1/2}-e^{2x}}{x^2}$

$=\lim\limits_{x\to 0}\frac{\left(1+\frac{1}{2}4x+\frac{1}{2}\cdot\left(\frac{1}{2}-1\right)\frac{1}{2!}16x^2+\ldots\right)-\left(1+\frac{2x}{1!}+\frac{4x^2}{2!}+\ldots\right)}{x^2}$

$= -4$

16. (b, c, d) : $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$

Applying $C_3 \to C_3 - \alpha C_1 - C_2$, we get

$$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ a\alpha+b & b\alpha+c & -(a\alpha^2+2b\alpha+c) \end{vmatrix}$$

$= (b^2 - ca)(a\alpha^2 + 2b\alpha + c) = 0$

if a, b, c are in G.P. or α is a root of $ax^2 + 2bx + c = 0$.

17. (a, b, d) : Let d be the common difference and $a_m = 2$, $a_n = 31$, then $(n - m)d = 29 \Rightarrow d = \frac{29}{n-m}$, which is rational.

Also $a_r - a_m = (r - m)d \quad\Rightarrow\quad a_r$ is rational.

As each term is rational, sum to any finite number of terms must be rational.

18. (b, c) : Here, $f(x) = x^3 - x^2 + 100x + 2002$

$\Rightarrow\quad f'(x) = 3x^2 - 2x + 100 > 0\ \forall\ x \in R$

$\therefore\quad f(x)$ is strictly increasing.

Therefore, $f\left(\frac{1}{2000}\right) > f\left(\frac{1}{2001}\right)$

Also, $f(x - 1) > f(x - 2)$ as $x - 1 > x - 2$ for $\forall\ x$.

19. (a, c) : The lines will be concurrent if $\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$

$\Rightarrow\quad p(rq - p^2) - q(q^2 - rp) + r(pq - r^2) = 0$

or $\quad p^3 + q^3 + r^3 - 3pqr = 0$

Also, $p^3 + q^3 + r^3 - 3pqr = (p + q + r)(p^2 + q^2 + r^2 - pq - qr - rp)$

$\therefore\quad p + q + r = 0$

[$\because$ $p^2 + q^2 + r^2 - pq - qr - rp = 0$ only if $p = q = r$ in this case all the three lines will be same, $x + y + 1 = 0$.]

20. (a, b) : The three points lie in the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ (intercept form) and area of $\Delta ABC=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$=\frac{1}{2}\left|(-a\hat{i}+b\hat{j})\times(-a\hat{i}+c\hat{k})\right|$

21. (d) : Since, roots of $f(x) = 0$ are equal in magnitude and opposite in sign.

$\therefore$ Sum of the roots = 0 and product of the roots < 0

Sum of the roots $= 0 \Rightarrow a = 0$ ...(i)

Product of the roots $< 0 \quad\Rightarrow\quad 5a^2 - 6a < 0$

$\Rightarrow\quad a \in \left(0, \frac{6}{5}\right)$ ...(ii)

From (i) and (ii), $\alpha \in \phi$

22. (b) : If one root of $f(x) = 0$ is zero, then product of the roots is 0

$\Rightarrow\quad 5a^2 - 6a = 0 \quad\Rightarrow\quad a = 0, \frac{6}{5}$

Hence, $a = \frac{6}{5}$

($\because\ a \neq 0$ as sum of the roots is non zero.)

23. (b) : Range of $f(x)$ is $[-5, \infty)$ for every real x or range of $f(x) + 5$ is $[0, \infty)$ for every real x.

Now $f(x) + 5 = x^2 - 4ax + 5a^2 - 6a + 5$

For range to be $[0, \infty)$, $D = 0$

$\Rightarrow \ 16a^2 - 4(5a^2 - 6a + 5) = 0$

$\Rightarrow \ a^2 - 6a + 5 = 0 \Rightarrow a = 5$ or 1

24. (d) : $\lim\limits_{x \to -1} \dfrac{f(x)}{(x+1)^3} = 1$ [Given]

$\Rightarrow \ f(-1) = f'(-1) = f''(-1) = 0, f'''(-1) = 6$

$\therefore \ f(x) = A(x+1)^4 + (x+1)^3$

$\Rightarrow \ f'''(x) = 24A(x+1) + 6$

$f'''(0) = 0 \Rightarrow 24A + 6 = 0 \Rightarrow A = -\dfrac{1}{4}$

$\therefore \ f(x) = (x+1)^3 - \dfrac{1}{4}(x+1)^4$

$\Rightarrow \ f'(x) = 3(x+1)^2 - (x+1)^3$

$\Rightarrow \ f''(x) = 6(x+1) - 3(x+1)^2$

$f'(x) = 0 \Rightarrow (x+1)^2(2-x) = 0$

$\Rightarrow \ x = -1, x = 2$ and $f''(2) = -9$

So, $x = 2$ is a point of maxima.

Thus, the maximum value is $f(2) = 27 - \dfrac{81}{4} = \dfrac{27}{4}$

25. (b) : $f(0) = 1 - \dfrac{1}{4} = \dfrac{3}{4}, f'(0) = 2$

Subnormal at $x = 0$ is $f(0)\,f'(0) = \dfrac{3}{4} \times 2 = \dfrac{3}{2}$

26. (c) : $f''(x) = 3(x+1)(2-x-1) = 3(x+1)(1-x)$ and $f'''(x) = -6x$

Now, $f''(x) = 0 \Rightarrow x = 1, -1$ and $f'''(x) \neq 0$ for $x = -1, 1$

27. (b) : The vector perpendicular to L_1 and L_2 is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{i} - 7\hat{j} + 5\hat{k}.$$

$\therefore$ Unit vector $(\hat{n})$ is $\dfrac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$.

28. (d) : The points on L_1 and L_2 are $A(-1, -2, -1)$ and $C(2, -2, 3)$

The shortest distance is

$$\overrightarrow{AC} \cdot \hat{n} = (3\hat{i} + 4\hat{k}) \cdot \frac{(-\hat{i} - 7\hat{j} + 5\hat{k})}{5\sqrt{3}} = \frac{17}{5\sqrt{3}}.$$

29. (c) : The $d.r.$'s of the normal are $-1, -7, 5$

$\therefore$ The plane is $-(x+1) - 7(y+2) + 5(z+1) = 0$

$\Rightarrow \ x + 7y - 5z + 10 = 0$

The distance of $(1, 1, 1)$ is $13/\sqrt{75} = 13/5\sqrt{3}$.

30. (c) : (A) Given, ${}^nC_0 + {}^nC_1 + {}^nC_2 = 46$

$\Rightarrow \ 1 + n + \dfrac{n(n-1)}{2} = 46$

$\Rightarrow \ 2 + 2n + n^2 - n = 92 \ \Rightarrow \ n^2 + n - 90 = 0$

$\Rightarrow \ (n-9)(n+10) = 0 \ \Rightarrow \ n = 9, n \neq -10$

$\therefore \ \left(x^2 + \dfrac{1}{x}\right)^n = \left(x^2 + \dfrac{1}{x}\right)^9$

$\therefore \ T_{r+1} = {}^9C_r (x^2)^{9-r} . \left(\dfrac{1}{x}\right)^r = {}^9C_r . x^{18-3r}$

Now, $18 - 3r = 0 \Rightarrow r = 6$

$\Rightarrow \ T_{r+1} = {}^9C_6 = {}^9C_3 = \dfrac{9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3} = 84$

(B) Here, $T_{r+1} = {}^nC_r \left(\dfrac{5}{x^2}\right)^{n-r} . (x^4)^r$

$= {}^nC_r \cdot (5)^{n-r} \cdot x^{-2n+2r+4r} = {}^nC_r \cdot (5)^{n-r} \cdot x^{6r-2n}$

$\because \ 6r - 2n = 0 \ \Rightarrow r = n/3 \ \therefore \ n = 3, 6, 9, \ldots..$

(C) Here, $T_{r+1} = {}^nC_r (x^3)^{n-r} . \left(-\dfrac{1}{x^2}\right)^r = {}^nC_r . (-1)^r . x^{3n-5r}$

For coefficient of x^5, we get $3n - 5r = 5$

$\Rightarrow \ r = \dfrac{3n-5}{5} = p(\text{say})$

For coefficient of x^{10}, we get $3n - 5r = 10$

$\Rightarrow \ r = \dfrac{3n-10}{5} = q(\text{say})$

$\because \ p - q = 1$ (using values of r) ...(i)

Also, ${}^nC_p(-1)^p + {}^nC_q(-1)^q = 0$

$\Rightarrow \ {}^nC_p(-1)^p + {}^nC_{p-1}(-1)^{p-1} = 0$ (From (i))

$\Rightarrow \ (-1)^p\{{}^nC_p - {}^nC_{p-1}\} = 0 \ \Rightarrow {}^nC_p - {}^nC_{p-1} = 0$

$\Rightarrow \ {}^nC_p = {}^nC_{p-1} \ \Rightarrow n = p + p - 1 \Rightarrow \ p = \dfrac{n+1}{2}$

$\therefore \ r = p \ \Rightarrow \ \dfrac{3n-5}{5} = \dfrac{n+1}{2}$

$\Rightarrow \ 6n - 10 = 5n + 5 \ \Rightarrow n = 15$

(D) We have, $(1 + x + x^2)^4$

$= (1 + x(1+x))^4 = 1 + {}^4C_1\, x(1+x) + {}^4C_2\, x^2(1+x)^2 + {}^4C_3\, x^3(1+x)^3 + {}^4C_4\, x^4(1+x)^4$

$= 1 + 4x(1+x) + 6x^2(1 + x^2 + 2x) + 4x^3(1 + x^3 + 3x + 3x^2) + x^4(x^4 + 4x^3 + 6x^2 + 4x + 1)$

$\therefore$ Coefficient of $x^4 = 6 + 12 + 1 = 19$

31. (d) : (A) $C_1 : x^2 + y^2 - 2a(x+y) + a^2 = 0$,

Centre : (a, a), radius : a,

$C_2 : x^2 + y^2 - 2b(x+y) + b^2 = 0$

Centre : (b, b), radius b

Since C_1 and C_2 touch each other

$\Rightarrow \sqrt{2}(b-a) = a + b \ \Rightarrow \dfrac{b}{a} = (\sqrt{2}+1)^2 = 3 + 2\sqrt{2}$

(B) C_1 and C_2 intersect orthogonally

$\Rightarrow 2(ab + ab) = a^2 + b^2 \Rightarrow b^2 - 4ab + a^2 = 0 \Rightarrow \dfrac{b}{a} = 2 + \sqrt{3}$

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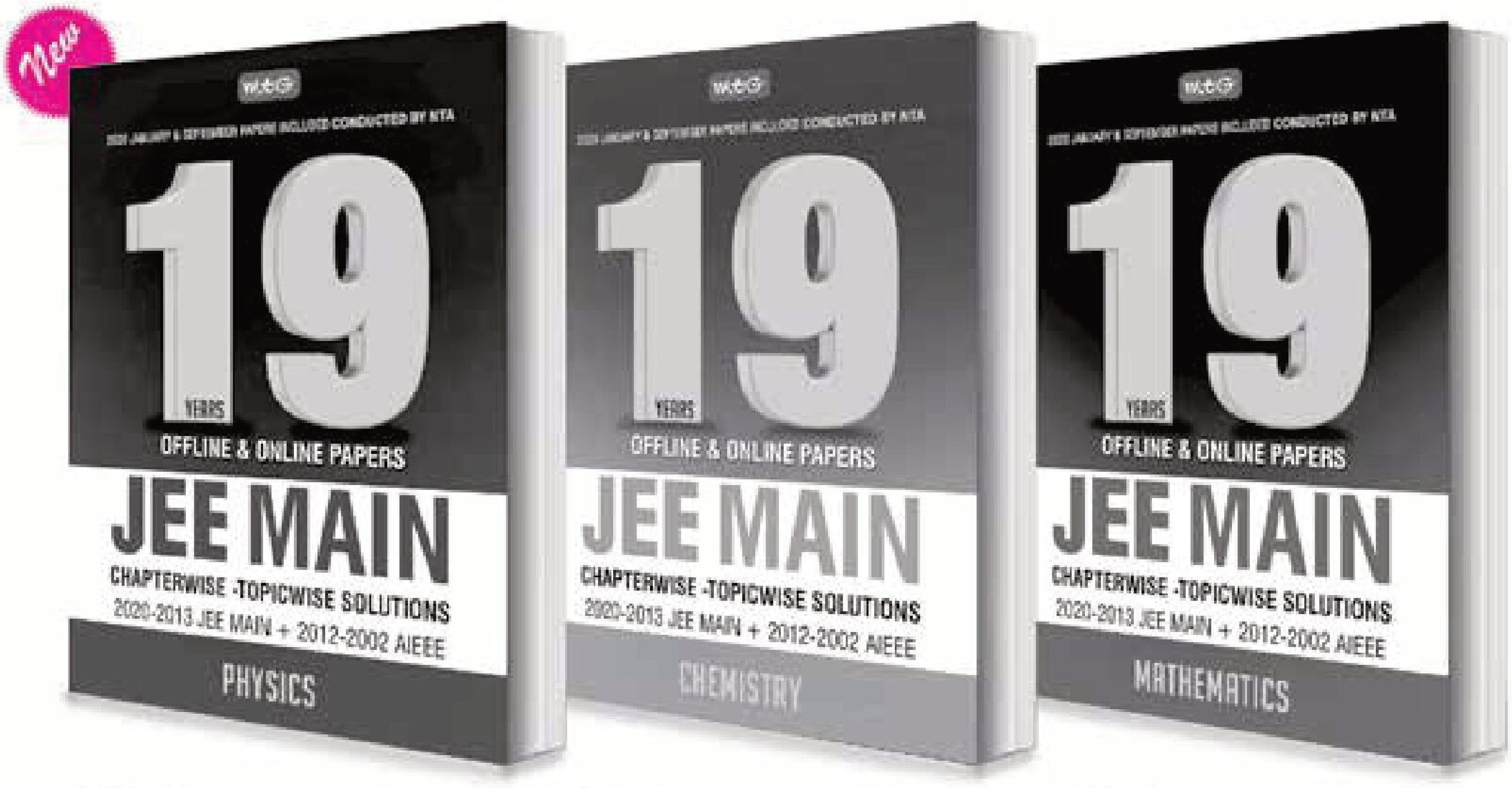
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(C) The common chord is

$2(b - a)(x + y) = b^2 - a^2$.

It passes through $(a, a) \Rightarrow b/a = 3$.

(D) C_2 passes through (a, a)

$$\Rightarrow 2a^2 - 4ab + b^2 = 0 \Rightarrow \frac{b}{a} = 2 + \sqrt{2}.$$

32. (a) : (A) The value of $\{x\} = x - 2$ for x close to 2 and $x > 2$

$$L = \lim_{x\to 2^+} \frac{(x-2)\sin(x-2)}{(x-2)^2} = \lim_{x\to 2^+} \frac{\sin(x-2)}{x-2} = 1.$$

(B) $\lim\limits_{x\to 2} [2 - x] + [x - 2] - x$

$= \lim\limits_{x\to 2} 2 + [-x] + [x] - 2 - x = \lim\limits_{x\to 2} [-x] + [x] - x$

R.H.L. $= \lim\limits_{x\to 2^+} [-x] + [x] - 2 = -3 + 2 - 2 = -3$

L.H.L. $= \lim\limits_{x\to 2^-} [-x] + [x] - 2 = -2 + 1 - 2 = -3$

$\therefore\ L = -3$.

(C) $\{2 - x\} + \{x - 2\} = 2 - x - [2 - x] + x - 2 - [x - 2]$
$= -[2 - x] - [x - 2] = -2 - [-x] - [x] + 2 = - [-x] - [x]$

R.H.L.$= \lim\limits_{x\to 2^+} - [-x] - [x] = 3 - 2 = 1$

L.H.L.$= \lim\limits_{x\to 2^-} - [-x] - [x] = 2 - 1 = 1$

$\therefore$ L = 1.

$$\textbf{(D)}\ \text{L} = \lim_{x\to 0^+} \frac{e^{\frac{1}{x}} - e^{\frac{-1}{x}}}{e^{\frac{1}{x}} + e^{\frac{-1}{x}}} = \lim_{x\to 0} \frac{1 - e^{\frac{-2}{x}}}{1 + e^{\frac{-2}{x}}} = \frac{1-0}{1+0} = 1$$

33. (4) : Putting $x = 1$ and $y = 1$, we get

$f^2(1) - f(1) - 6 = 0 \Rightarrow f(1) = 3$ or $f(1) = -2$

Now, putting $y = 1$, we get

$$f(x)\cdot f(1) = f(x) + 2\left(\frac{1}{x} + 2\right) = f(x) + 2\left(\frac{2x+1}{x}\right)$$

$$\Rightarrow\ f(x)[f(1) - 1] = \frac{2(2x+1)}{x} \ \Rightarrow\ f(x) = \frac{2(2x+1)}{x[f(1)-1]}$$

For $f(1) = 3,\ f(x) = \dfrac{(2x+1)}{x}$ ...(i)

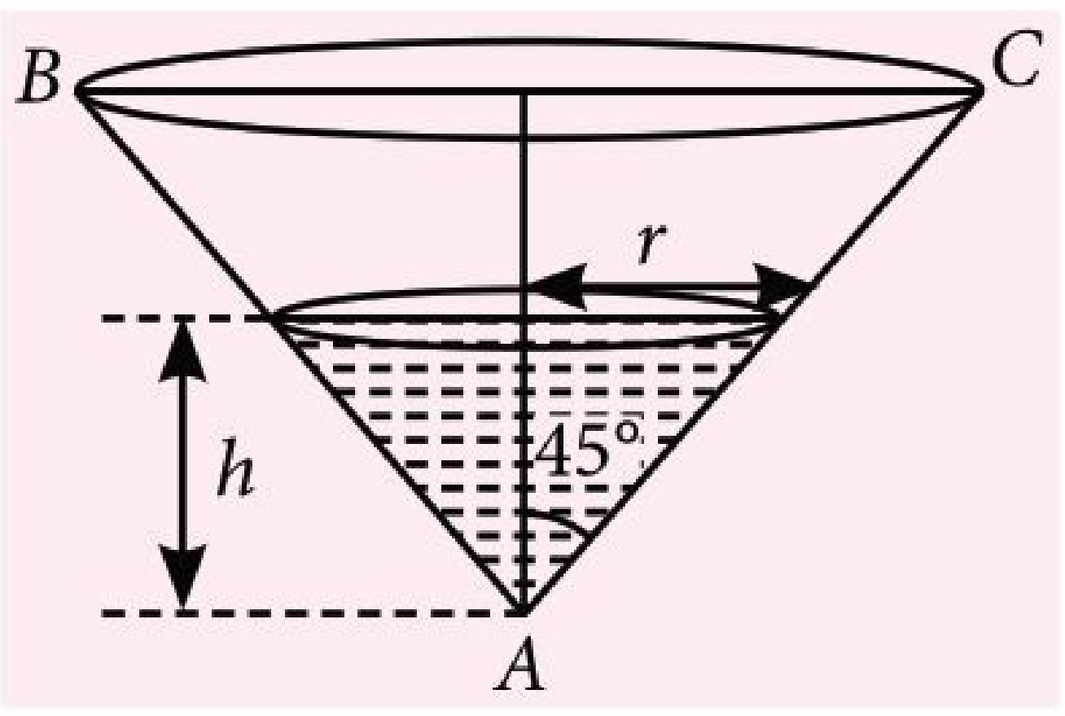
and for $f(1) = -2,\ f(x) = \dfrac{2(2x+1)}{-3x}$ ...(ii)

$$\Rightarrow\ f\left(\frac{1}{2}\right) = 4$$

34. (5) : We have,

$$\frac{dV}{dt} = 2 \text{ or } \frac{d}{dt}\left(\frac{1}{3}\pi r^3\right) = 2$$

[$\because r = h$ as $\theta = 45°$]



$$\Rightarrow \pi r^2 \frac{dr}{dt} = 2 \Rightarrow \frac{dr}{dt} = \frac{2}{\pi r^2} \quad \text{...(i)}$$

Now, perimeter $= 2\pi r = p$ (say)

$$\therefore\ \frac{d}{dt}(2\pi r) = 2\pi\frac{dr}{dt} = 2\pi\left(\frac{2}{\pi r^2}\right) = \frac{4}{r^2} \quad \text{...(ii) [Using (i)]}$$

When $h = 2$ m, $r = 2$ m

Hence, $d = \dfrac{dp}{dt} = \dfrac{4}{4} = 1$ m/s.

35. (8) : Let $\alpha = \sin^{-1}\dfrac{\sqrt{5}}{3}$, then $\cos^{-1}\dfrac{\sqrt{5}}{3} = \dfrac{\pi}{2} - \alpha$

Hence the expression $= \sin\left\{2\left(\alpha - \dfrac{\pi}{2} + \alpha\right)\right\}$

$= \sin(4\alpha - \pi) = -\sin 4\alpha = -4\sin\alpha\cos\alpha(1 - 2\sin^2\alpha)$

$$= (-4)\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)\left(\frac{-1}{9}\right) = \frac{8\sqrt{5}}{81}$$

36. (1) : As $f(-1) = f(1)$ and Roll's theorem is not applicable, then it implies $f(x)$ is either discontinuous or $f'(x)$does not exist at atleast one point in $(-1, 1)$. $g(x) = 0$ at atleast one value of x in $(-1, 1)$. Thus $k = 1$

37. (104) : The required number of points $= {}^8C_2 \times 1 + {}^4C_2 \times 2 + ({}^8C_1 \times {}^4C_1) \times 2 = 28 + 12 + 32 \times 2 = 104$.

38. (8) : $I = \int (x^9 + x^6 + x^3)(2x^6 + 3x^3 + 6)^{1/3}\, dx$

$= \int x^2(x^6 + x^3 + 1)(2x^9 + 3x^6 + 6x^3)^{1/3} dx$

Put $2x^9 + 3x^6 + 6x^3 = t, 18(x^8 + x^5 + x^2)dx = dt$

$\Rightarrow\ 18x^2(x^6 + x^3 + 1)dx = dt$

$$\therefore\ I = \frac{1}{24}(2x^9 + 3x^6 + 6x^3)^{4/3} + K \quad \therefore\ \frac{AB}{4} = 8$$

39. (15) : $np = 2$ ...(i), $npq = 1$ ...(ii)

$$\Rightarrow\ q = \frac{1}{2},\ p = \frac{1}{2},\ n = 4 \text{ (from (i) and (ii))}$$

$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$$= 1 - {}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16}$$

40. (9) : $\vec{r} \times \vec{b} = \vec{c} \times \vec{b} \Rightarrow (\vec{r} - \vec{c}) \times \vec{b} = \vec{0}$

$$\Rightarrow \vec{r} = \vec{c} + \lambda\vec{b} \quad \text{...(i)}$$

$$\Rightarrow\ \vec{r}\cdot\vec{a} = \vec{a}\cdot\vec{c} + \lambda\vec{a}\cdot\vec{b} \Rightarrow \vec{r} = \vec{c} - \frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\vec{b}$$

$$\Rightarrow\ \vec{r}\cdot\vec{b} = \vec{b}\cdot\vec{c} - \frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\left(|\vec{b}|^2\right) = 1 - \frac{-4}{1}(2) = 9$$

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MOCK TEST PAPER

ISI

Indian Statistical Institute

*ALOK KUMAR

PART A

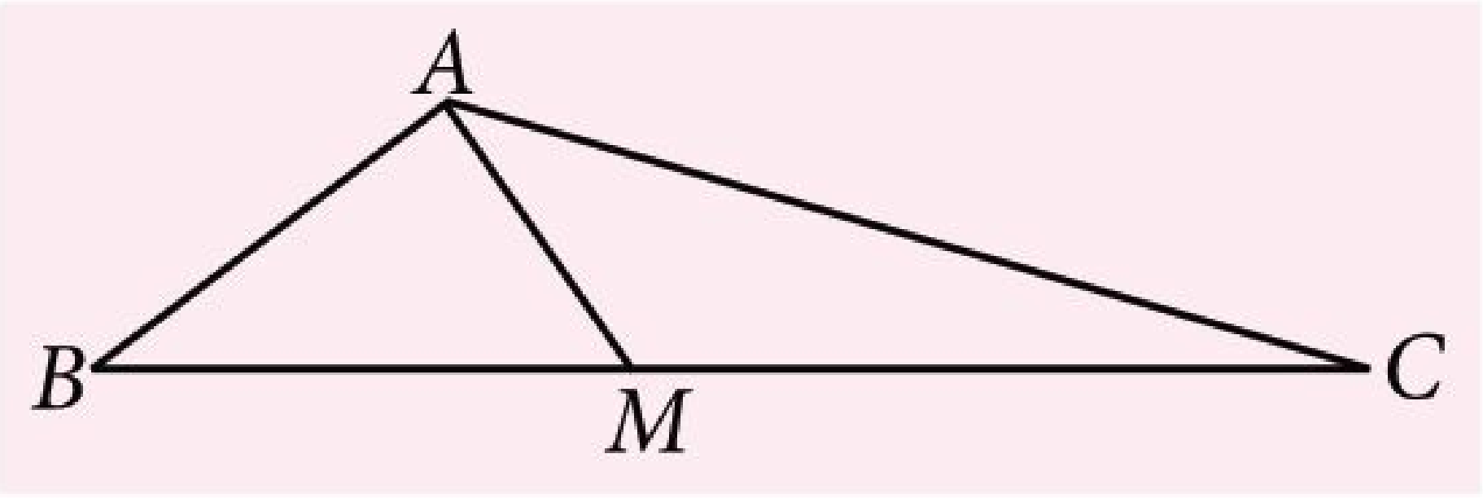
1. If $i^2 = -1$, then $(1 + i)^{20} - (1 - i)^{20}$ equals

(a) -1024 (b) $-1024i$
(c) 0 (d) 1024
(e) $1024i$

2. A fair die is rolled six times. The probability of rolling at least five at least five times is

(a) $\frac{13}{729}$ (b) $\frac{12}{729}$ (c) $\frac{2}{729}$ (d) $\frac{3}{729}$
(e) None of these

3. In the adjoining figure triangle ABC is such that $AB = 4$ and $AC = 8$. If M is the midpoint of BC and $AM = 3$, what is the length of BC?



(a) $2\sqrt{26}$ (b) $2\sqrt{31}$
(c) 9 (d) $4 + 2\sqrt{13}$
(e) Not enough information given to solve the problem

4. Ann and Barbara were comparing their ages and found that Barbara is as old as Ann was when Barbara was as old as Ann had been when Barbara was half as old as Ann is . If the sum of their present ages is 44 years, then Ann's age is

(a) 22 (b) 24 (c) 25 (d) 26
(e) 28

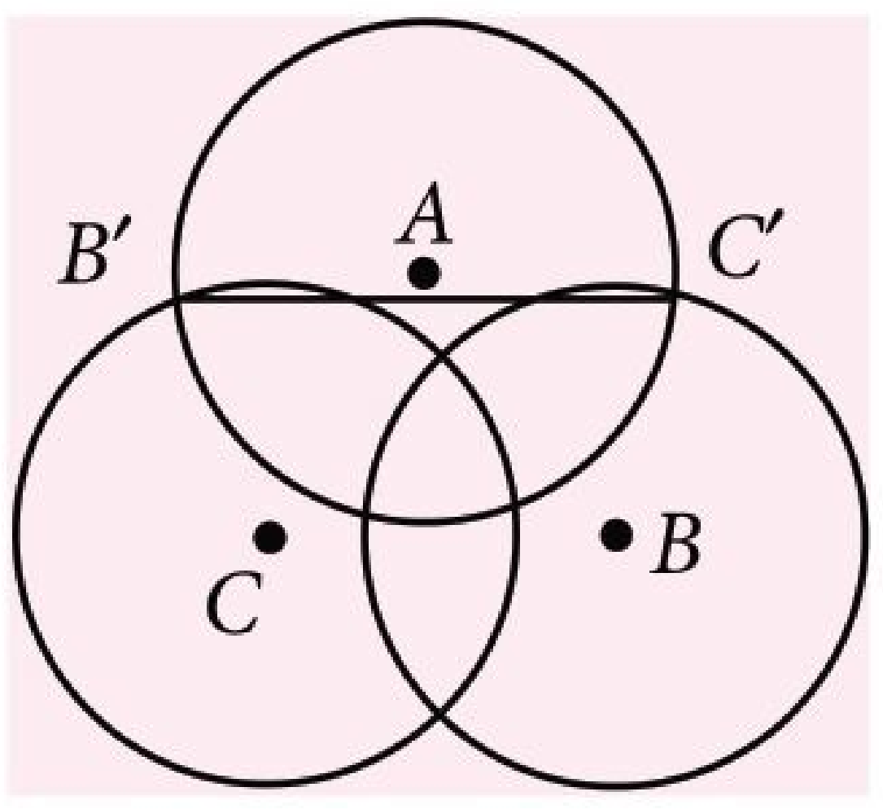
5. In a tennis tournament, n women and $2n$ men play, and each player plays exactly one match with every other player. If there are no ties and the ratio of the number of matches won by women to the number of matches won by men is 7/5, then n equals

(a) 2 (b) 4 (c) 6 (d) 7
(e) None of these

6. An ordered pair (b, c) of integer, each of which has absolute value less than or equal to five, is chosen at random, with each such ordered pair having an equal likelihood of being chosen. What is the probability that the equation $x^2 + bx + c = 0$ will not have distinct positive real roots ?

(a) $\frac{106}{121}$ (b) $\frac{108}{121}$ (c) $\frac{110}{121}$ (d) $\frac{112}{121}$
(e) None of these

7. Circles with centers A, B and C each have radius r, where $1 < r < 2$. The distance between each pair of centers is 2. If B' is the point of intersection of circle A and circle C which is outside circle B, and if C' is the point of intersection of circle A and circle B which is outside circle C, then length $B'C'$ equals



(a) $3r - 2$ (b) r^2
(c) $r+\sqrt{3(r-1)}$ (d) $1+\sqrt{3(r^2-1)}$
(e) None of these

8. A box contains 2 pennies, 4 nickels and 6 dimes. Six coins are drawn without replacement, with each coin having an equal probability of being chosen. What is the probability that the value of the coins drawn is at least 50 cents?

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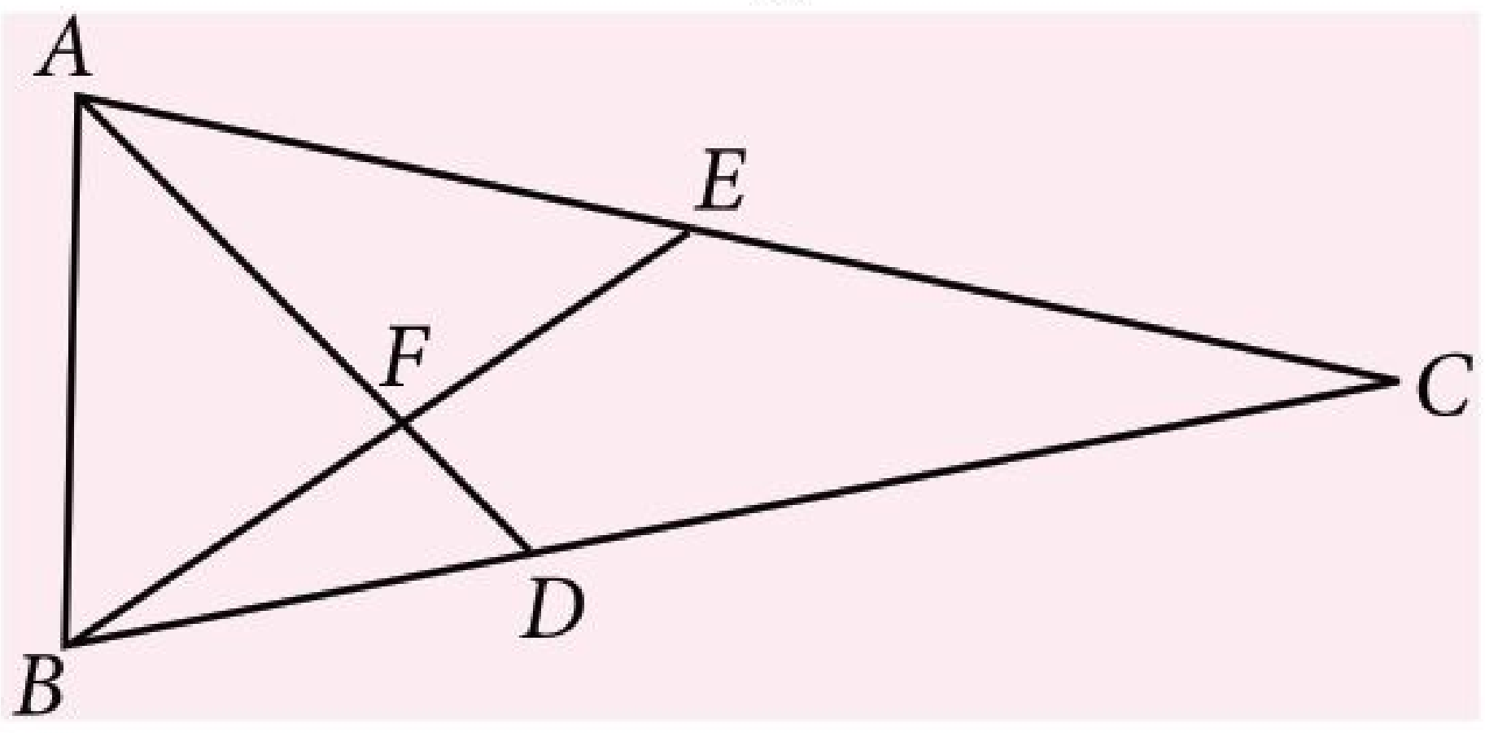
(a) $\frac{37}{924}$ (b) $\frac{91}{924}$ (c) $\frac{127}{924}$ (d) $\frac{132}{924}$

(e) None of these

9. In triangle ABC, $\angle CBA = 72°$, E is the midpoint of side AC, and D is a point on side BC such that $2BD = DC$; AD and BE intersect at F. The ratio of the area of ΔBDF to the area of quadrilateral $FDCE$ is



(a) 1/5 (b) 1/4 (c) 1/3 (d) 2/5

(e) None of these

10. A six digit number (base 10) is squarish if it satisfies the following conditions:

(i) None of its digits is zero;

(ii) It is a perfect square; and

(iii) The first two digits, the middle two digits and the last two digits of the number are all perfect squares when considered as two digit numbers.

How many squarish numbers are there?

(a) 0 (b) 2 (c) 3 (d) 8

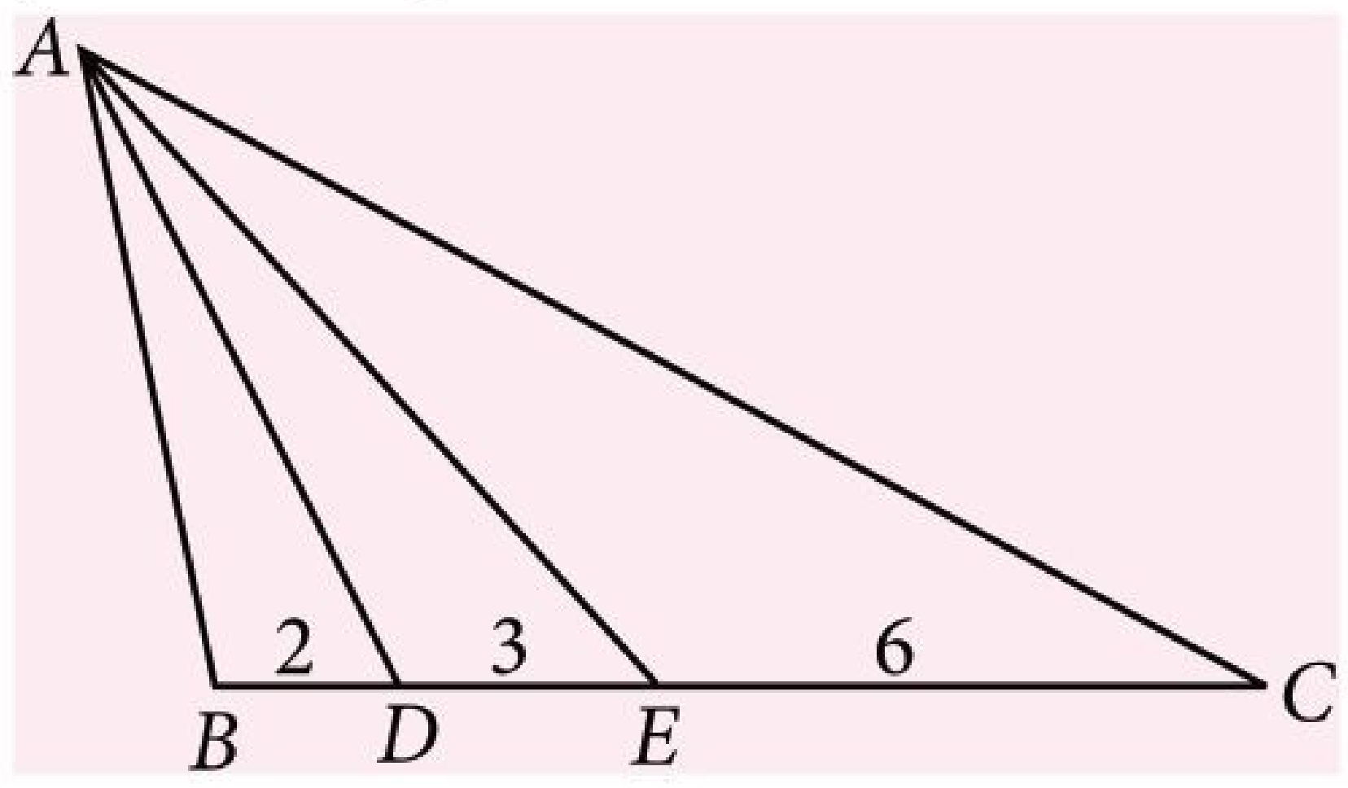
(e) 9

11. How many lines in a three-dimensional rectangular coordinate system pass through four distinct points of the form (i, j, k), where i, j and k are positive integers not exceeding four?

(a) 60 (b) 64 (c) 72 (d) 76

(e) 100

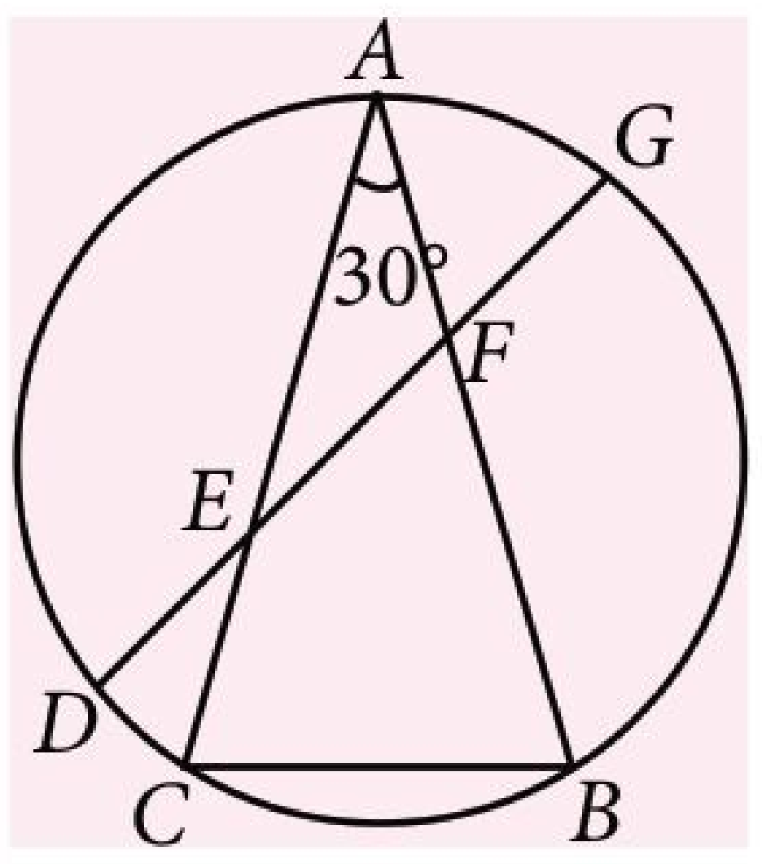
12. In triangle ABC in the adjoining figure, AD and AE trisect $\angle BAC$. The lengths of BD, DE and EC are 2, 3, and 6, respectively. The length of the shortest side of ΔABC is



(a) $2\sqrt{10}$ (b) 11 (c) $6\sqrt{6}$ (d) 6

(e) Not uniquely determined by the given information

13. In the adjoining figure triangle ABC is inscribed in a circle. Point D lies on $\overset{\frown}{AC}$ with $\overset{\frown}{DC} = 30°$, and point G lies on $\overset{\frown}{BA}$ with $\overset{\frown}{BG} > \overset{\frown}{GA}$. Side AB and side AC each have length equal to the length of chord DG, and $\angle CAB = 30°$. Chord DG intersects sides AC and AB at E and F, respectively. The ratio of the area of ΔAFE to the area of ΔABC is



(a) $\frac{2-\sqrt{3}}{3}$ (b) $\frac{2\sqrt{3}-3}{3}$

(c) $7\sqrt{3}-12$ (d) $3\sqrt{3}-5$

(e) $\frac{9-5\sqrt{3}}{3}$

14. Consider the set of all equations :
$x^3 + a_2x^2 + a_1x + a_0 = 0$, where a_2, a_1, a_0 are real constants and $|a_i| \le 2$ for $i = 0, 1, 2$. Let r be the largest positive real number which satisfies at least one of these equations. Then

(a) $1 \le r < \frac{3}{2}$ (b) $\frac{3}{2} \le r < 2$

(c) $2 \le r < \frac{5}{2}$ (d) $\frac{5}{2} \le r < 3$

(e) $3 \le r < \frac{7}{2}$

15. The lengths of the sides of a triangle are consecutive integers, and the largest angle is twice the smallest angle. The cosine of the smallest angle is

(a) 3/4 (b) 7/10 (c) 2/3 (d) 9/14

(e) None of these

16. Suppose $z = a + ib$ is a solution of the polynomial equation $c_4z^4 + ic_3z^3 + c_2z^2 + ic_1z + c_0 = 0$, where c_0, c_1, c_2, c_3, c_4, a and b are real constants and $i^2 = -1$. Which one of the following must also be a solution ?

(a) $-a-ib$ (b) $a-ib$

(c) $-a+ib$ (d) $b+ia$

(e) None of these

17. A set of consecutive positive integers beginning with 1 is written on a blackboard. One number is erased. The average (arithmetic mean) of the remaining numbers is $35\frac{7}{17}$. What number was erased?

(a) 6 (b) 7 (c) 8 (d) 9

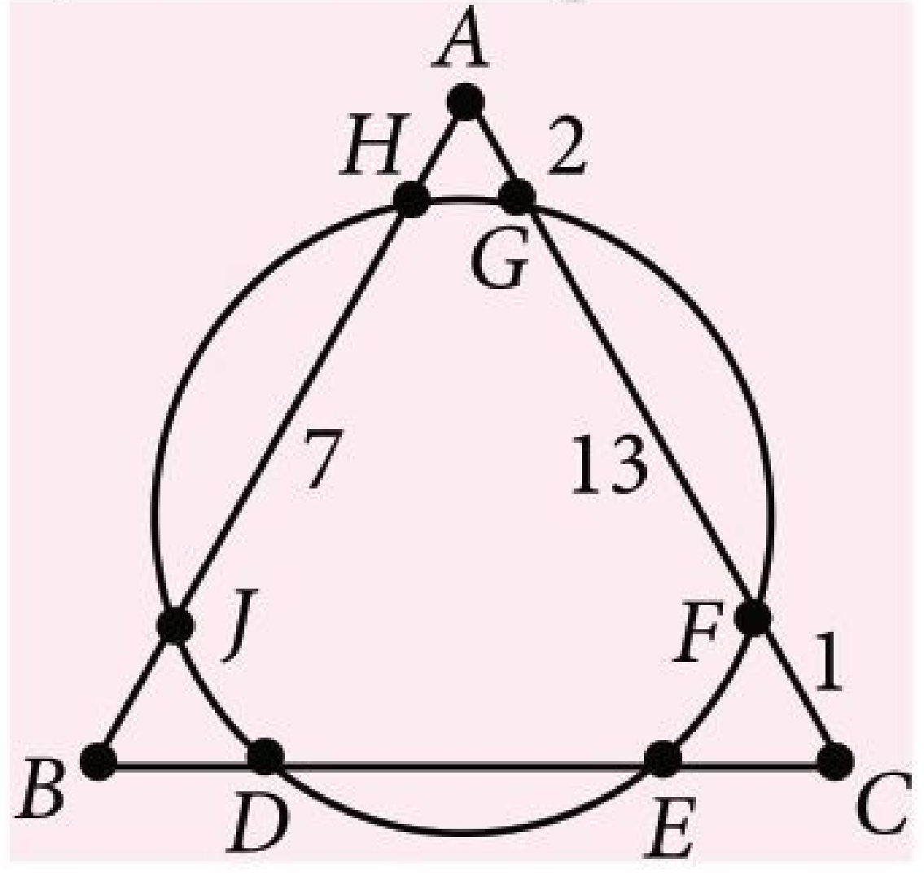
(e) Can not be determined

18. Let x, y and z be three positive real numbers whose sum is 1. If no one of these numbers is more than twice any other, then the minimum possible value of the product xyz is

(a) $\frac{1}{32}$ (b) $\frac{1}{36}$ (c) $\frac{4}{125}$ (d) $\frac{1}{127}$

(e) None of these

19. In the adjoining figure, the circle meets the sides of an equilateral triangle at six points. If $AG = 2$, $GF = 13$, $FC = 1$ and $HJ = 7$, then DE equals



(a) $2\sqrt{22}$ (b) $7\sqrt{3}$ (c) 9 (d) 10

(e) 13

PART B

1. If a, b, c are the sides of a triangle and $a + b + c = 2$, prove that $a^2 + b^2 + c^2 + 2abc < 2$.

2. For any $x \in R$ and $n \in N$, define

$x^{(0)} = 1$ and $x^{(n)} = x(x - 1) \ldots. (x - n + 1)$.

Show that $(x+y)^{(n)} = \sum_{k=0}^{n} \begin{pmatrix} x \\ k \end{pmatrix} x^{(k)} y^{(n-k)}$

3. Find all real x, y satisfying $x^3 + y^3 = 7$ and $x^2 + y^2 + x + y + xy = 4$.

4. Determine the set of all positive integers n for which 3^{n+1} divides $2^{3^n} + 1$. Prove that 3^{n+2} does not divide $2^{3^n} + 1$ for any positive integer n.

5. Prove that the product of 4 consecutive natural numbers cannot be a cube.

6. Suppose $ABCD$ is a convex quadrilateral and P, Q are the midpoints of CD, AB. Let AP, DQ meet in X and BP, CQ meet in Y. Prove that $[ADX] + [BCY] = [PXQY]$. How does the conclusion alter if $ABCD$ is not a convex quadrilateral ?

7. Triangle ABC has incentre I and the incircle touches BC, CA at D, E respectively . If BI meets DE in G, show that AG is perpendicular to BG.

8. Suppose $ABCD$ is a quadrilateral such that a semicircle with its centre at the midpoint of AB and bounding diameter lying on AB touches the other three sides BC, CD and DA. Show that $AB^2 = 4BC \cdot AD$.

9. Suppose $ABCD$ is a rectangle and P, Q, R, S are points on the sides AB, BC, CD, DA respectively. Show that $PQ + QR + RS + SP \geq \sqrt{2}\ AC$.

10. How many increasing 3-term geometric progressions can be obtained from the sequence 1, 2, $2^2, 2^3, \ldots, 2^n$? (*e.g.*, $\{2^2, 2^5, 2^8\}$ is a 3-term geometric progression for $n \geq 8$.)

11. For any natural number n, $(n \geq 3)$, let $f(n)$ denote the number of non-congruent integer-sided triangles with perimeter n (*e.g.*, $f(3) = 1$, $f(4) = 0$, $f(7) = 2$). Show that

(i) $f(1999) > f(1996)$

(ii) $f(2000) = f(1997)$

SOLUTION

PART A

1. (c) : 1st Solution: Since $i^2 = -1$, $(1 + i)^2 = 2i$ and $(1 - i)^2 = -2i$. Writing

$$(1 + i)^{20} - (1 - i)^{20} = ((1 + i)^2)^{10} - ((1 - i)^2)^{10}$$

we have,

$$(1 + i)^{20} - (1 - i)^{20} = (2i)^{10} - (-2i)^{10} = 0$$

Observe that

$$i(1 - i) = i - i^2 = 1 + i, \text{ and } i^4 = (-1)^2 = 1$$

Therefore, $(1 + i)^4 = i^4(1 - i)^4 = (1 - i)^4$,

and $(1 + i)^{4n} - (1 - i)^{4n} = 0$ for $n = 1, 2, 3, \ldots.$

2nd Solution:

$$(1+i)^{20} = \left(\sqrt{2}\right)^{20} [\cos 900° + i \sin 900°] = -1024,$$

$$(1-i)^{20} = \left(\sqrt{2}\right)^{20} [\cos(-900°) + i \sin(-900°)] = -1024$$

and the difference is 0.

2. (a) : Let A be the event of rolling at least five; then the probability of A is $\frac{2}{6} = \frac{1}{3}$. In six rolls of a die the probability of event A happening six times is $\left(\frac{1}{3}\right)^6$. The probability of getting exactly five successes and one failure of A in a specific order is $\left(\frac{1}{3}\right)^5 . \frac{2}{3}$. Since there are six ways to do this, the probability of getting five successes and one failure of A in any order is

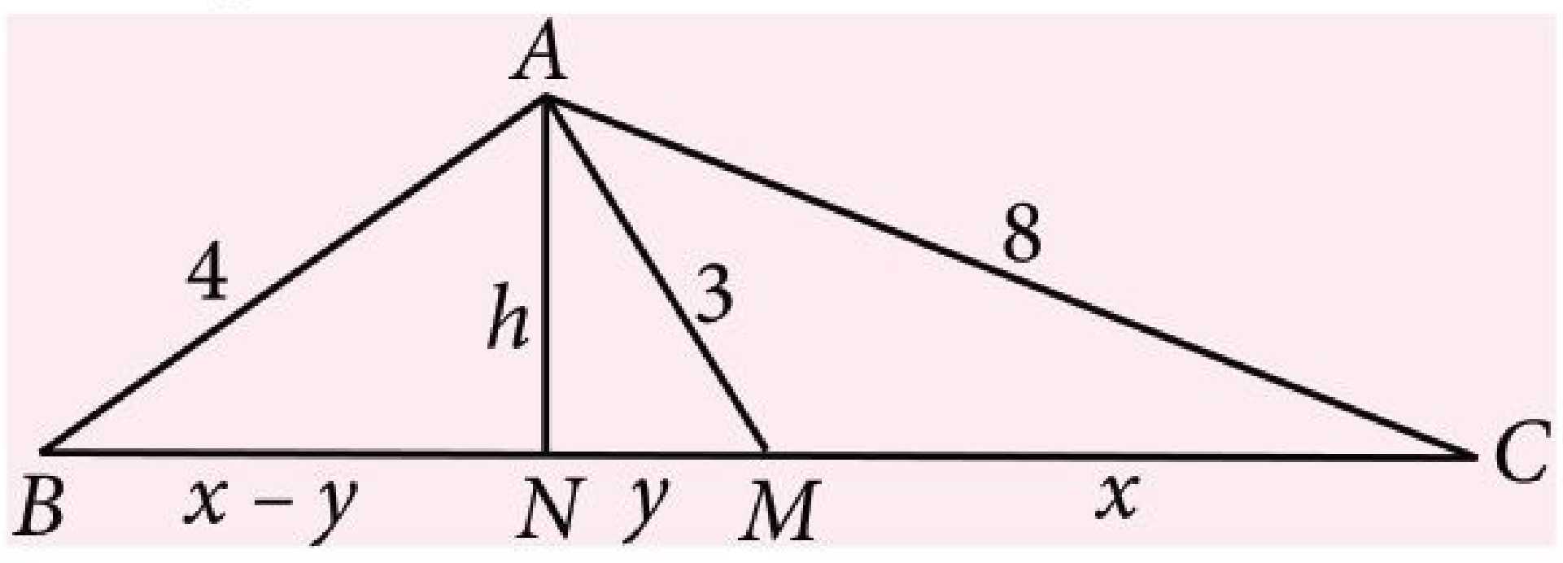
$$6\left(\frac{1}{3}\right)^5 . \frac{2}{3} = \frac{12}{729}.$$

The probability of getting all successes or five successes and one failure in any order is thus

$$\frac{12}{729} + \left(\frac{1}{3}\right)^6 = \frac{13}{729}.$$

3. (b) : 1st Solution: In the adjoining figure, let h be the length of altitude AN drawn to BC, let $x = BM$ and let $y = NM$. Then

$$h^2 + (x + y)^2 = 64,$$
$$h^2 + y^2 = 9$$
$$h^2 + (x - y)^2 = 16.$$



Subtracting twice the second equation from the sum of the first and third equations yield $2x^2 = 62$. Thus $x=\sqrt{31}$ and $BC=2\sqrt{31}$.

2nd Solution: Recall that the sum of the squares of the sides of a parallelogram is equal to the sum of the squares of its diagonals. Applying this to the parallelogram having AB and AC as adjacent sides yield $2(4^2 + 8^2) = 6^2 + (2x)^2$, $x=\sqrt{31}$.

4. (b) : The table below shows the ages of Ann and Barbara at various times referred to in the problem. The first column indicates their present ages. The second column shows their ages when Barbara was half as old as Ann is now – that was $y-\left(\frac{x}{2}\right)$ years ago, hence Ann's age was $x-\left(y-\frac{x}{2}\right)$ or $\frac{3x}{2}-y$. The third column refers to the time when Barbara was as old as Ann had been when Barbara was half as old as Ann is that was $y-\left(\frac{3x}{2}-y\right)$ or $2y-\frac{3x}{2}$ years ago; hence Ann's age then was $x-\left(2y-\frac{3x}{2}\right)$ or $\frac{5x}{2}-2y$.

Ann	x	$\frac{3x}{2}-y$	$\frac{5x}{2}-2y$
Barbara	y	$\frac{x}{2}$	$\frac{3x}{2}-y$

By the conditions stated in the problem, $x + y = 44$ and $y=\frac{5x}{2}-2y$; the simultaneous solution of these yield $x = 24$.

5. (c) : 1st Solution: Let $7m$, $5m$ be the total number of matches won by women and men, respectively. Now there are $\frac{n(n-1)}{2}$ matches between women, hence won by women.

There are $\frac{2n(2n-1)}{2}=n(2n-1)$ matches between men, hence won by men. Finally, there are $2n \cdot n = 2n^2$ mixed matches, of which $k=7m-\frac{n(n-1)}{2}$ are won by women, and $2n^2 - k = 5m - n(2n - 1)$ by men.

We note for later use that $k \le 2n^2$. Then

$$\frac{7m}{5m}=\frac{\frac{1}{2}n(n-1)+k}{n(2n-1)+2n^2-k}=\frac{1}{2}\frac{n(n-1)+2k}{4n^2-n-k}$$

$$\Rightarrow\ 5n(n - 1) + 10k = 14(4n^2 - n - k)$$

$$\Rightarrow\ 51n^2 - 9n - 24k = 3(17n^2 - 3n - 8k)= 0,$$

$$\Rightarrow\ 17n^2 - 3n = 8k,\ \frac{17n^2-3n}{8}=k.$$

Since $k \le 2n^2$, it follows that $17^2 - 3n \le 16n^2$, so $n(n - 3) \le 0$, $n \le 3$. Since k is an integer, $n \ne 1$ or 2; hence $n = 3$.

2nd Solution: The total number of games played was

$$\binom{3n}{2}=\frac{3n(3n-1)}{2}$$

Suppose that $7m$ games were won by women, and $5m$ games were won by men. Then

$7m+5m=12m=\frac{3n(3n-1)}{2}$ and $3n(3n - 1) = 24m$.

Since m is an integer, $3n(3n - 1)$ is a multiple of 24. But this is not true for $n = 2, 4, 6, 7$. (It is true for $n = 3$.)

6. (e) : There are six statements equivalent for integers b, c with absolute value at most 5:

1) the equation $x^2 + bx + c = 0$ has positive roots;

2) the equation $x^2 + bx + c = 0$ has real roots, the smaller of which is positive;

3) $\sqrt{b^2-4c}$ is real and $-b-\sqrt{b^2-4c}>0$;

4) $0 \le b^2 - 4c < b^2$ and $b < 0$;

5) $0<c\le\frac{b^2}{4}$ and $b<0$;

6) $b = -2$ and $c = 1$; or $b = -3$ and $c = 1$ or 2; or $b = -4$ and $c = 1, 2, 3$ or 4; or $b = -5$ and $c = 1, 2, 3, 4$ or 5.

The roots corresponding to the pairs (b, c) described in (6) will be distinct unless $b^2 = 4c$. Thus, deleting $(b, c) = (-2, 1)$ and $(-4, 4)$ from the list in (6) yields the ten pairs resulting in distinct positive roots.

The desired probability is then $1-\frac{10}{11^2}=\frac{111}{121}$.

7. (d) : Denote by A' the analogous intersection point of the circles with centers at B and C, so that, by symmetry, $A'B'C'$ and ABC are both equilateral triangles. Again, by symmetry, ΔABC and $\Delta A'B'C'$ have a common centroid; call it K. Let M be the midpoint of the line segment BC. From the triangle $A'BC$ we see that the length $A'M = \sqrt{r^2 - 1}$. Since corresponding lengths in similar triangles are proportional,

$$\frac{B'C'}{BC} = \frac{A'K}{AK}.$$

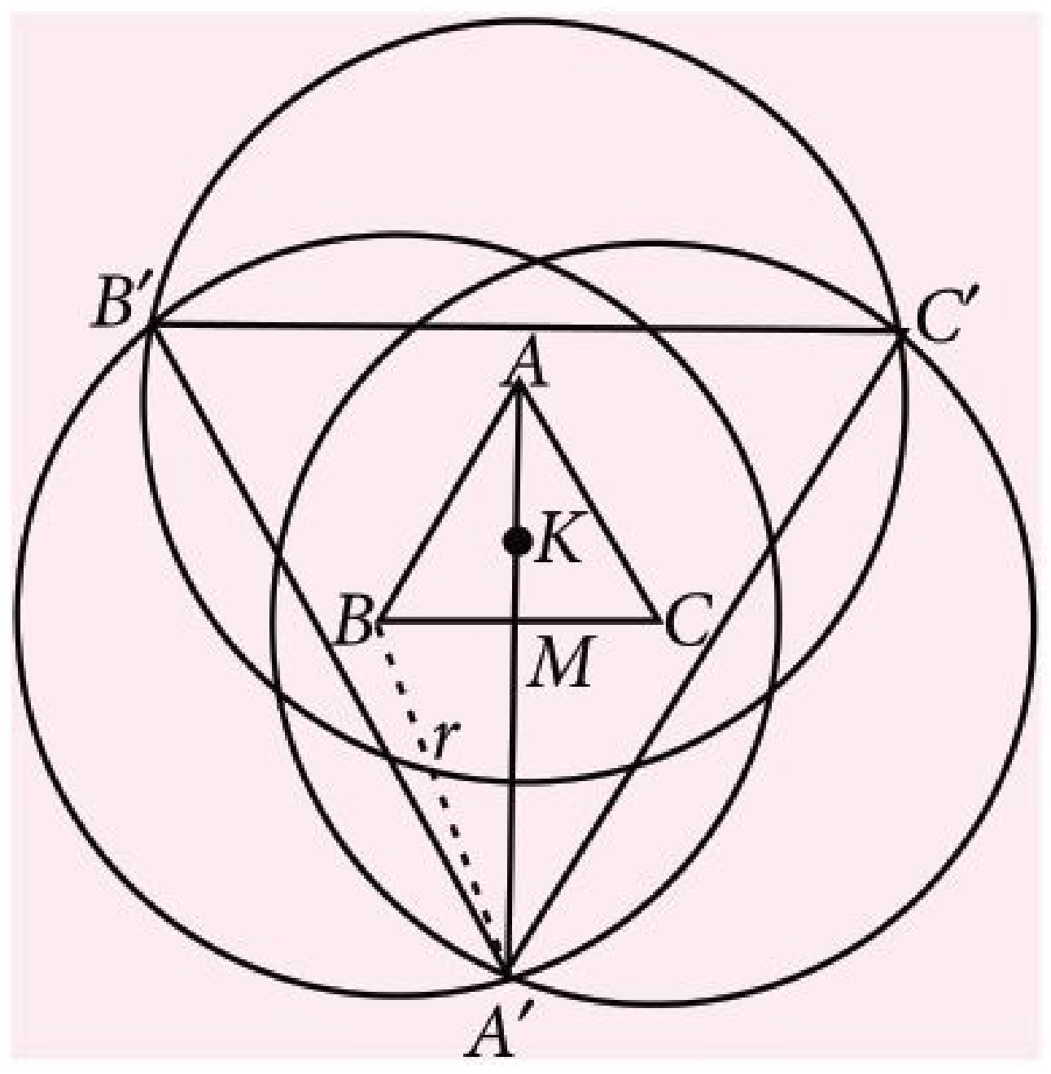
Since equilateral ΔABC has sides of length 2, we find that $B'C' = 2\frac{A'K}{AK}$ and also that altitude AM has length $\sqrt{3}$.

Consequently,

$$AK = \frac{2}{3}AM = \frac{2}{3}\sqrt{3},\ MK = \frac{1}{3}AM = \frac{1}{3}\sqrt{3}$$

and $A'K = A'M + MK = \sqrt{r^2 - 1} + \left(\frac{\sqrt{3}}{3}\right)$

Thus, $B'C' = 2\dfrac{\sqrt{r^2-1} + \left(\frac{\sqrt{3}}{3}\right)}{2\left(\frac{\sqrt{3}}{3}\right)} = \sqrt{3(r^2 - 1)} + 1$.



8. (c) : The number of ways of choosing 6 coins from 12 is $\binom{12}{6} = 924$

[The symbol $\binom{n}{k}$ denotes the number of ways k things may be selected from a set of n distinct objects.] "Having at least 50 cents" will occur if one of the following cases occurs:

1) Six dimes are drawn.
2) Five dimes and any other coin are drawn.
3) Four dimes and two nickels are drawn.

The number of ways 1), 2) and 3) can occur are $\binom{6}{6}, \binom{6}{5}\binom{6}{1}$ and $\binom{6}{4}\binom{4}{2}$, respectively. The desired probability is, therefore,

$$\frac{\binom{6}{6} + \binom{6}{4}\binom{4}{2} + \binom{6}{5}\binom{6}{1}}{924} = \frac{127}{924}.$$

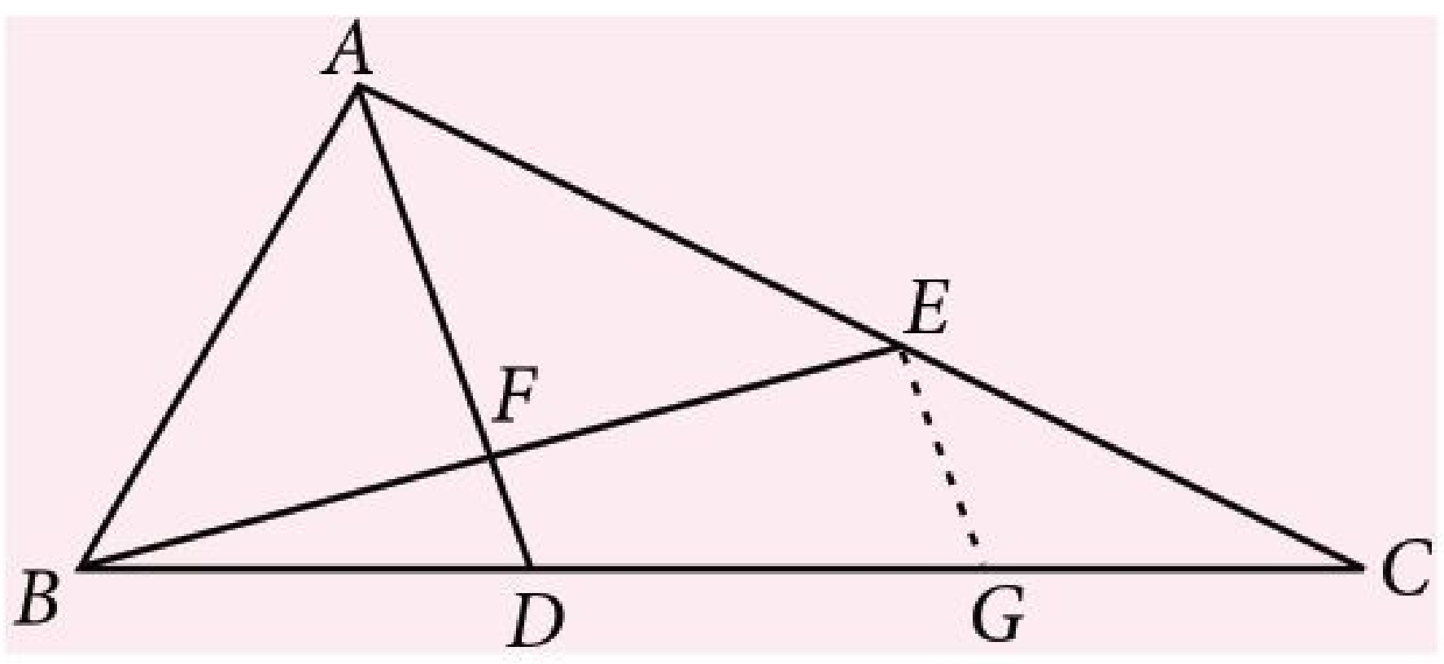
9. (a) : In the adjoining figure the line segment from E to G, the midpoint of DC, is drawn. Then

$$\text{area } \Delta EBG = \left(\frac{2}{3}\right)(\text{area } \Delta EBC)$$

$$\text{area } \Delta BDF = \left(\frac{1}{4}\right)(\text{area } \Delta EBG) = \left(\frac{1}{6}\right)(\text{area } \Delta EBC)$$

(Note that since EG connects the midpoints of sides AC and DC in ΔACD, EG is parallel to AD.) Therefore,

$$\text{area } FDCE = \left(\frac{5}{6}\right)(\text{area } \Delta EBC)$$



and $\dfrac{\text{area } \Delta BDF}{\text{area } FDCE} = \dfrac{1}{5}$.

The measure of $\angle CBA$ was not needed.

10. (b) : That N is squarish may be expressed algebraically as follows: there are single digit integers A, B, C, a, b, c such that

$$N = 10^4A^2 + 10^2B^2 + C^2 = (10^2a + 10b + c)^2,$$

where each of A, B, C exceeds 3, and so a and c are positive. Since $10^2B^2 + C^2 < 10^4$, we can write

$$10^4A^2 < (10^2a + 10b + c)^2 < 10^4A^2 + 10^4 < 10^4(A + 1)^2$$

Taking square roots we obtain

$$100A < 100a + 10b + c < 100A + 100,$$

from which it follows that $A = a$. Hence $a \geq 4$. Now consider

$$M = N - 10^4A^2 = (10^2a + 10b + c)^2 - 10^4a^2$$
$$= 10^3(2ab) + 10^2(b^2 + 2ac) + 10(2bc) + c^2.$$

Since M has only four digits, $2ab < 10$, which implies that $ab \leq 4$. Thus either (i) $b = 0$, or (ii) $a = 4$ and $b = 1$. In case (ii),

$$N = (410 + c)^2 = 168100 + 820c + c^2$$

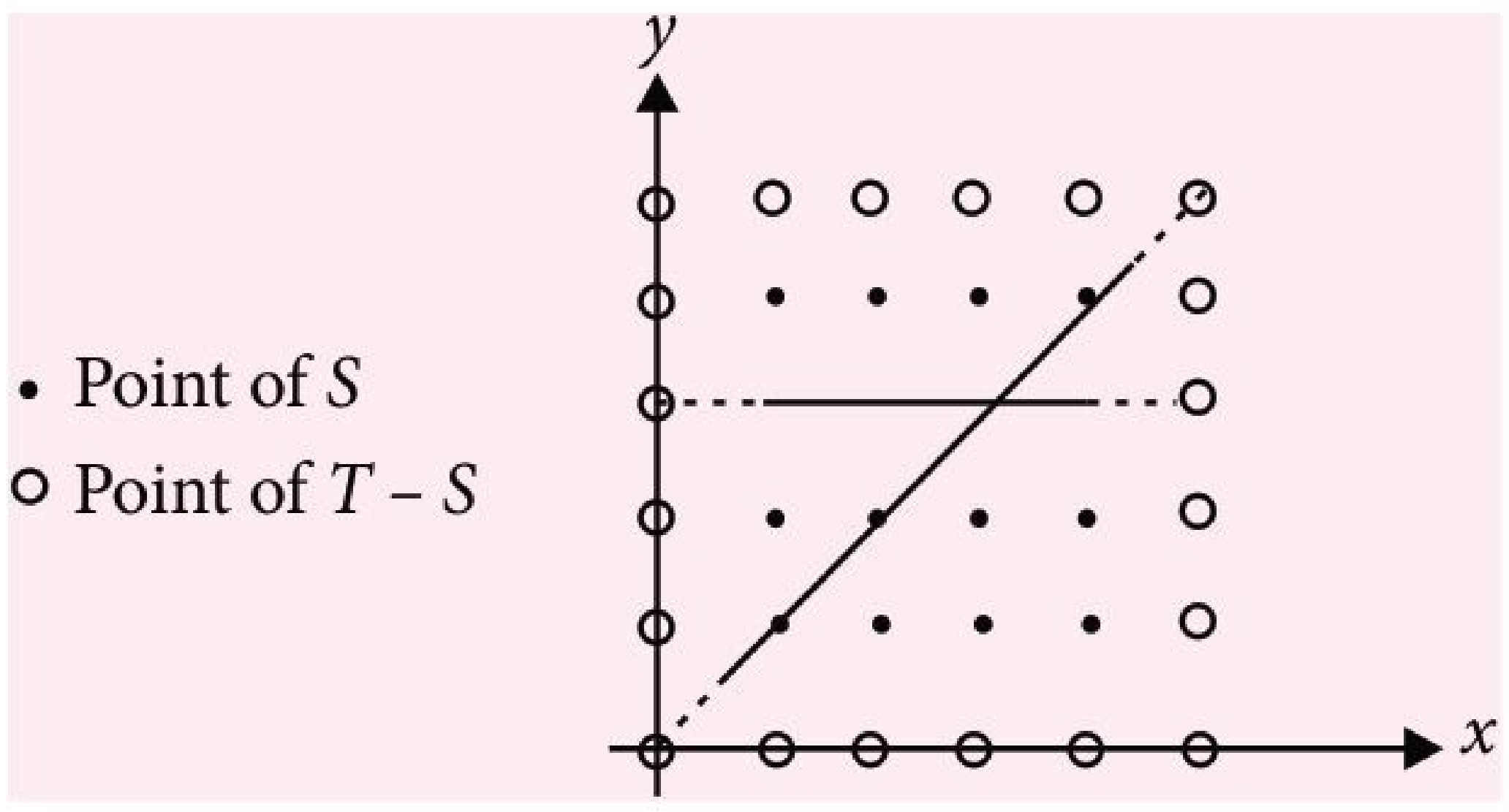
If $c = 1$ or 2, the middle two digits of N form a number exceeding 81, hence not a square. If $c \geq 3$, then the leftmost two digits of N are 17. Therefore case (i) must hold, and we have

$$N = (10^2a + c)^2 = 10^4a^2 + 10^2(2ac) + c^2$$

Thus $a \geq 4$, $c \geq 4$ and $2ac$ is an even two-digit perfect square. It is now easy to check that either $a = 8$, $c = 4$, $N = 646416$, or $a = 4$, $c = 8$, $N = 166464$.

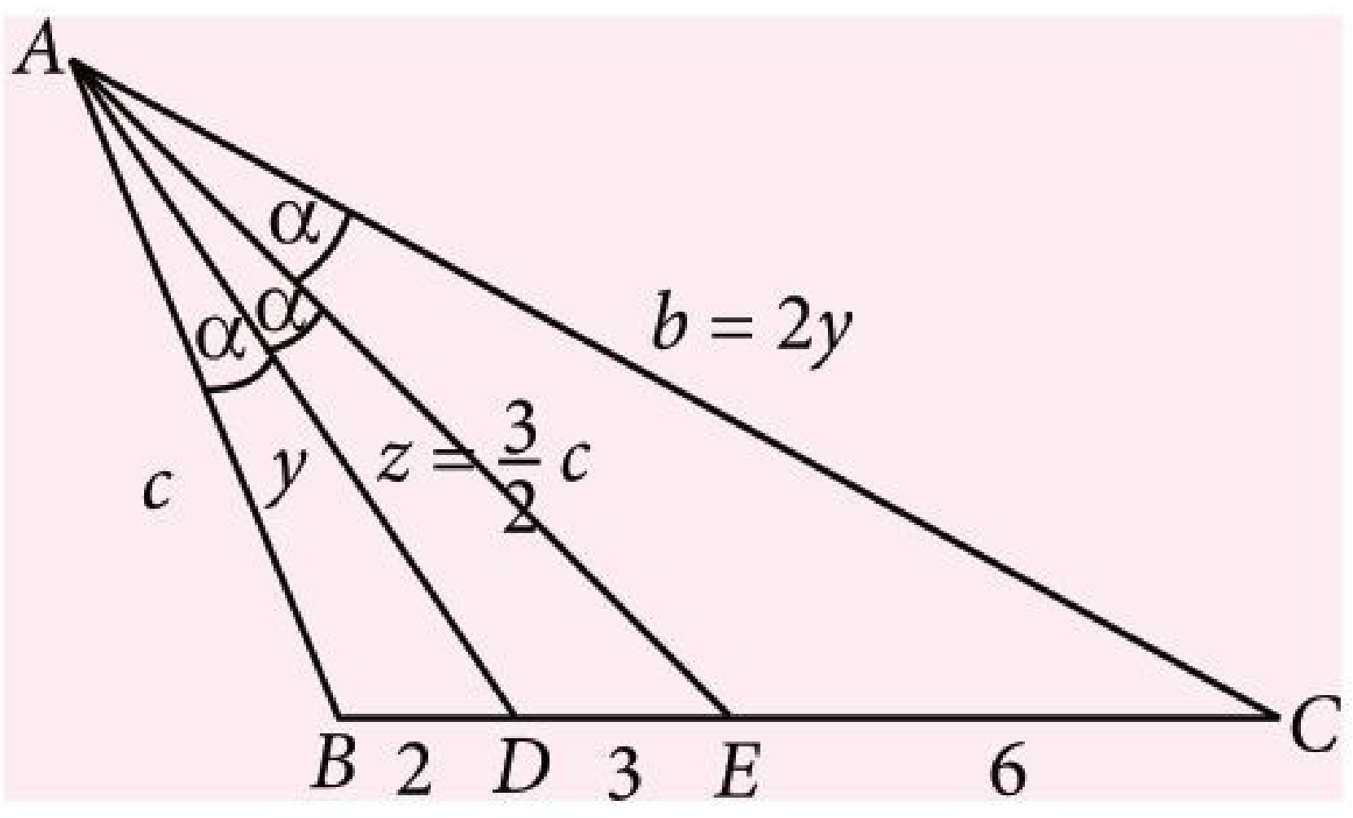
11. (d) : 1st Solution: Consider the smallest cube containing all the lattice points (i, j, k), $1 \le i, j, k \le 4$, in a three dimensional Cartesian coordinate system. There are 4 main diagonals. There are 24 diagonal lines parallel to a coordinate plane: 2 in each of four planes parallel to each of the three coordinate planes. There are 48 lines parallel to a coordinate axis: 16 in each of the three directions. Threfore, there are
$4 + 24 + 48 = 76$ lines.

2nd Solution: Let S be the set of lattice points (i, j, k) with $1 \le i, j, k \le 4$, and let T be the set of lattice points (i, j, k) with $0 \le i, j, k \le 5$. Every line segment containing four points of S can be extended at both ends so as to contain six points of T.



12. (a) : 1st Solution: In the adjoining figure let $\angle BAC = 3\alpha$, $c = AB$, $y = AD$, $z = AE$ and $b = AC$. Then by the angle bisector theorem

$$1)\quad \frac{c}{z} = \frac{2}{3} \text{ and } \frac{y}{b} = \frac{1}{2}; \text{ so } z = \frac{3}{2}c,\ b = 2y.$$



Using the law of cosines in ΔADB, ΔAED and ΔACE, respectively, yields the following expressions for $\cos\alpha$.

$$\frac{c^2+y^2-4}{2cy} = \frac{\frac{9}{4}c^2+y^2-9}{3cy} = \frac{\frac{9}{4}c^2+4y^2-36}{6cy}$$

The equality of the first and second expressions implies $3c^2 - 2y^2 = 12$.

The equality of the first and third expressions implies $3c^2 - 4y^2 = -96$.

Solving these two equations for c^2 and y^2 yields

$$c^2 = 40,\ y^2 = 54$$

Thus the sides of the triangle are

$$AB = c = 2\sqrt{10} \approx 6.3$$

$$AC = b = 2y = 2\sqrt{54} = 6\sqrt{6} \approx 14.7,$$

$$BC = 11$$

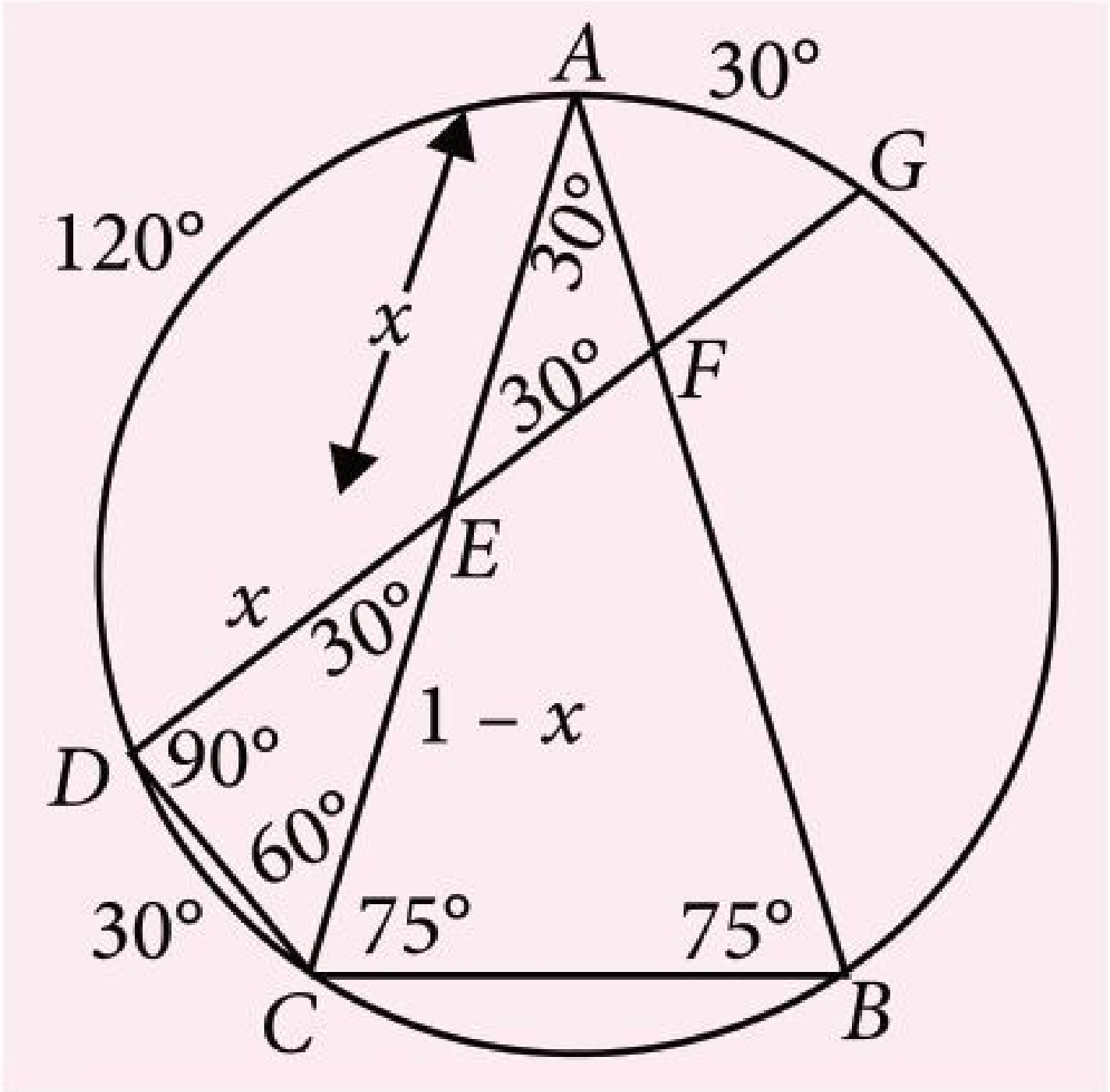
2nd Solution: Using the angle bisector formula and (1) above, we have

$$y^2 + 6 = cz = \frac{2}{3}z^2,\ z^2 + 18 = yb = 2y^2$$

Solving these equations for y^2 and z^2 yields $y^2 = 54$, $z^2 = 90$. Therefore,

$$AB = c = \frac{2}{3}\sqrt{90} = 2\sqrt{10},\ AC = b = 2\sqrt{54} = 6\sqrt{6}.$$

13. (c) : In the figure, line segment DC is drawn. Since $\widehat{AC} = 150°$, $\widehat{AD} = \widehat{AC} - \widehat{DC} = 150° - 30° = 120°$. Hence $\angle ACD = 60°$. Since $AC = DG$, $\widehat{GA} = \widehat{GD} - \widehat{AD} = \widehat{AC} - 120° = 30°$. Therefore $\widehat{CG} = 180°$ and $\angle CDG = 90°$. So ΔDEC is a $30° - 60° - 90°$ triangle.



Since we are looking for the ratio of the areas, let us assume without loss of generality that $AC = AB = DG = 1$. Since AC and DG are chords of equal length in a circle, we have $AE = DE$. Let x be their common length. Then $CE = 1 - x = \frac{2x}{\sqrt{3}}$. Solving for x yields $AE = x = 2\sqrt{3} - 3$. Since BA and DG are chords of equal length in a circle, we have $FG = FA$, and since ΔFAE is isosceles, $EF = FA$. Thus

$$EF = FG = \frac{1}{2}(1-x)$$

Therefore,

$$\text{area } \Delta AFE = \frac{1}{2}(AE)(AF)\sin 30°$$

$$= \frac{1}{2}x\frac{1-x}{2}.\frac{1}{2} = \frac{x-x^2}{8} = \frac{7\sqrt{3}-12}{4}$$

$$\text{area } \Delta ABC = \frac{1}{2}(AB)(AC)\sin 30° = \frac{1}{4}$$

Hence, $\dfrac{\text{area } \Delta AFE}{\text{area } \Delta ABC} = 7\sqrt{3} - 12$

14. (d) : Let $g(x) = x^3 + a_2x^2 + a_1x + a_0$ be an arbitrary cubic with constants of the specified form. Because x^3 dominates the other terms for large enough x,

$g(x) > 0$ for all x greater than the largest real root of g. Thus we seek a particular g in which the terms $a_2x^2 + a_1x + a_0$ "hold down" $g(x)$ as much as possible, so that the value of the largest real root is as large as possible. This suggests that the answer to the problem is the largest root of $f(x) = x^3 - 2x^2 - 2x - 2$. Call this root r_0. Since $f(0) = -2$, r_0 is certainly positive. To verify this conjecture, note that for $x \geq 0$,

$$-2x^2 \leq a_2x^2,\ -2x \leq a_1x, \text{ and } -2 \leq a_0.$$

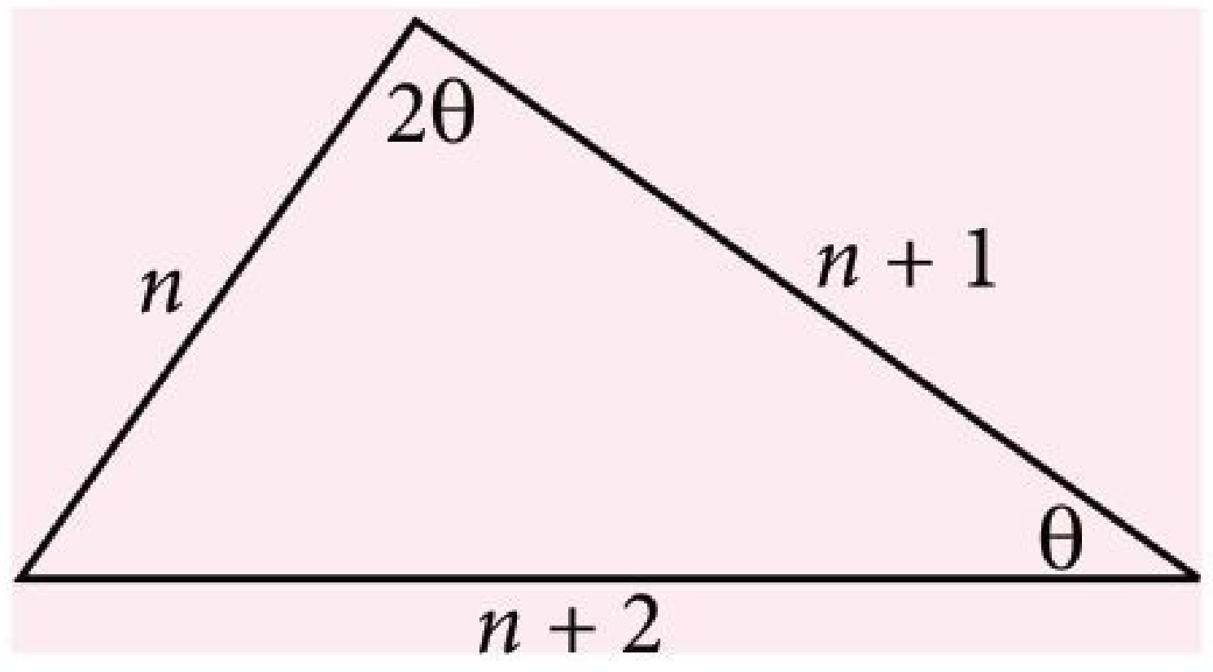
Summing these inequalities and adding x^3 to both sides yields $f(x) \leq g(x)$ for all $x \geq 0$. Thus for all $x > r_0$, $0 < f(x) \leq g(x)$. That is, no g has a root larger than r_0, so r_0 is the r of the problem.

A sketch of f shows that it has a typical cubic shape, with largest root a little less than 3. In fact, $f(2) = -6$ and $f(3) = 1$. To be absolutely sure the answer is (d), not (c), compute $f\left(\frac{5}{2}\right)$ to see if it is negative. Indeed,

$$f\left(\frac{5}{2}\right) = -\frac{31}{8}.$$

15. (a) : In the adjoining figure, n denotes the length of the shortest side, and θ denotes the measure of the smallest angle. Using the law of sines and writing $2\sin\theta\cos\theta$ for $\sin 2\theta$, we obtain

$$\frac{\sin\theta}{n} = \frac{2\sin\theta\cos\theta}{n+2},\quad \cos\theta = \frac{n+2}{2n}$$



Equating $\frac{n+2}{2n}$ to the expression of $\cos\theta$ obtained from the law of consines yields

$$\frac{n+2}{2n} = \frac{(n+1)^2 + (n+2)^2 - n^2}{2(n+1)(n+2)}$$

$$= \frac{(n+1)(n+5)}{2(n+1)(n+2)} = \frac{n+5}{2(n+2)}$$

Thus $n = 4$ and $\cos\theta = \frac{4+2}{4(2)} = \frac{3}{4}$

16. (c) : We recall the theorem that complex roots of polynomials with real coefficients come in conjugate pairs. Though not applicable to the given polynomial, that theorem is proved by a technique which we can use to work this problem too. Namely, conjugate both sides of the original equation

$$0 = c_4z^4 + ic_3z^3 + c_2z^2 + ic_1z + c_0$$

obtaining

$$0 = c_4\bar{z}^4 - ic_3\bar{z}^3 + c_2\bar{z}^2 - ic_1\bar{z} + c_0$$

$$= c_4(-\bar{z})^4 + ic_3(-\bar{z})^3 + c_2(-\bar{z})^2 - ic_1(-\bar{z}) + c_0$$

That is, $-\bar{z} = -a+ib$ is also a solution of the original equation. (One may check by example that neither $-a - ib$ nor $a - ib$ nor $b + ia$ need be a solution.) For instance, consider the equation $z^4 - iz^3 = 0$ and the solution $z = i$. Here $a = 0$, $b = 1$. Neither $-i$ nor 1 is a solution. [Alternatively, the substitution $z = i\omega$ into the given equation makes the coefficients real and the above quoted theorem applicable.]

17. (b) : Let n be the last number on the board. Now the largest average possible is attained if 1 is erased; the average is then

$$\frac{2+3+\ldots+n}{n-1} = \frac{\frac{(n+1)n}{2}-1}{n-1} = \frac{n+2}{2}$$

The smallest average possible is attained when n is erased; the average is then

$$\frac{n(n-1)}{2(n-1)} = \frac{n}{2}$$

Thus, $\frac{n}{2} \leq 35\frac{7}{17} \leq \frac{n+2}{2}$, $n \leq 70\frac{14}{17} \leq n+2$,

$$68\frac{14}{17} \leq n \leq 70\frac{14}{17}.$$

Hence $n = 69$ or 70. Since $35\frac{7}{17}$ is the average of $(n - 1)$ integers, $\left(35\frac{7}{17}\right)(n-1)$ must be an integer and n is 69. If x is the number erased, then

$$\frac{\frac{1}{2}(69)(70)-x}{68} = 35\frac{7}{17}$$

So $69\cdot 35 - x = \left(35\frac{7}{17}\right)68 = 35\cdot 68 + 28$

$$35 - x = 28 \quad \text{or} \quad x = 7.$$

18. (a) : Let $m = x_0y_0z_0$ be the minimum value, and label the numbers so that $x_0 \leq y_0 \leq z_0$. In fact $z_0 = 2x_0$, for if $z_0 < 2x_0$, then by decreasing x_0 slightly, increasing z_0 by the same amount, and keeping y_0 fixed, we would get new values which still meet the constraints but which have a smaller product–contradiction! To show this contradiction formally, let $x_1 = x_0 - h$ and $z_1 = z_0 + h$, where $h > 0$ is so small that $z_1 \leq 2x_1$ also. Then x_1, y_0, z_1 also meet all the original constraints, and

$$x_1y_0z_1 = (x_0 - h)y_0(z_0 + h)$$

$$= x_0y_0z_0 + y_0[h(x_0 - z_0) - h^2] < x_0y_0z_0$$

So $z_0 = 2x_0$, $y_0 = 1 - x_0 - z_0 = 1 - 3x_0$, and

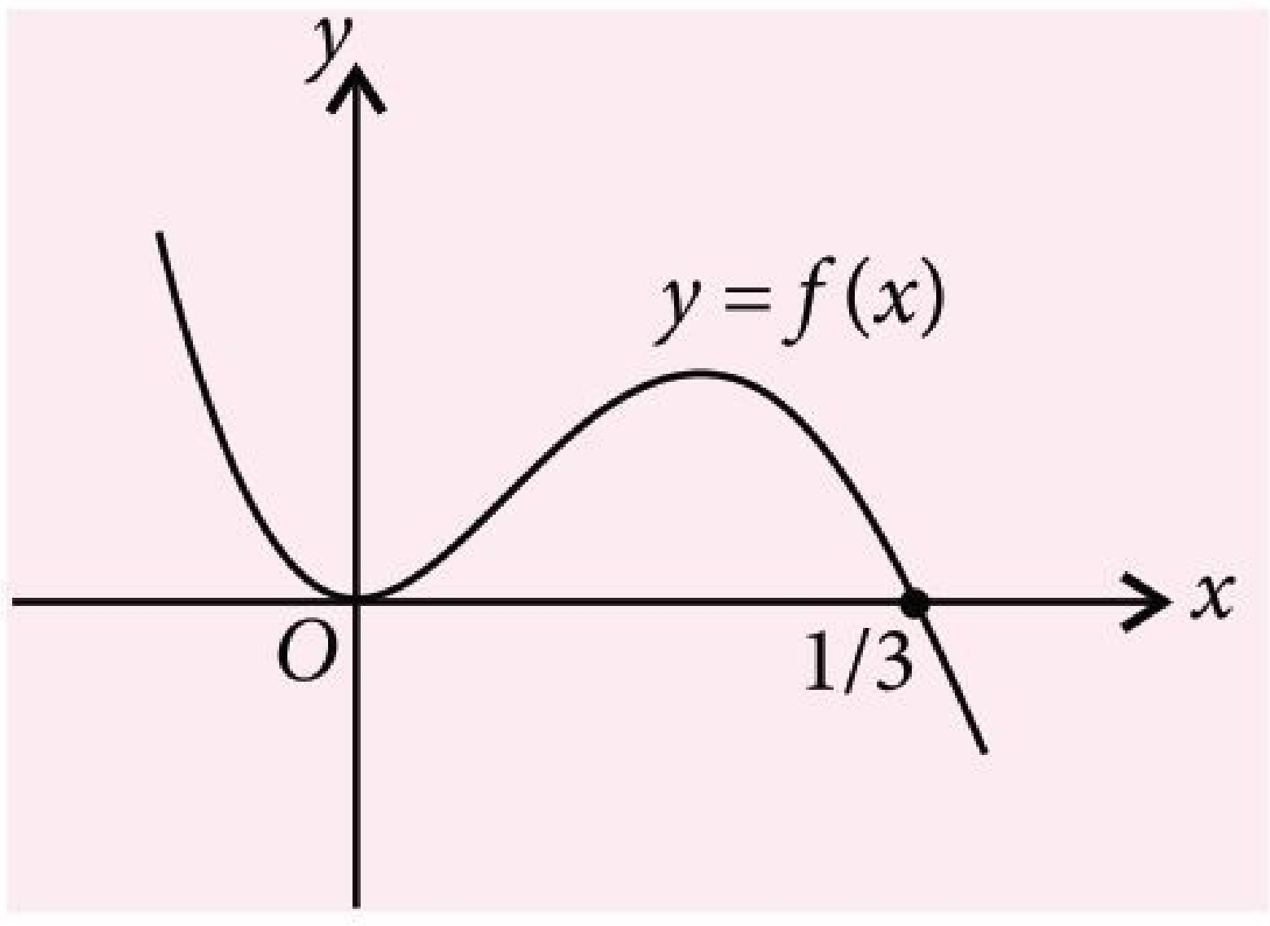
$$m = 2x_0^2(1 - 3x_0)$$

Also, $x_0 \le 1 - 3x_0 \le 2x_0$, or equivalently, $\frac{1}{5} \le x_0 \le \frac{1}{4}$.

Thus m may be viewed as a value of the function

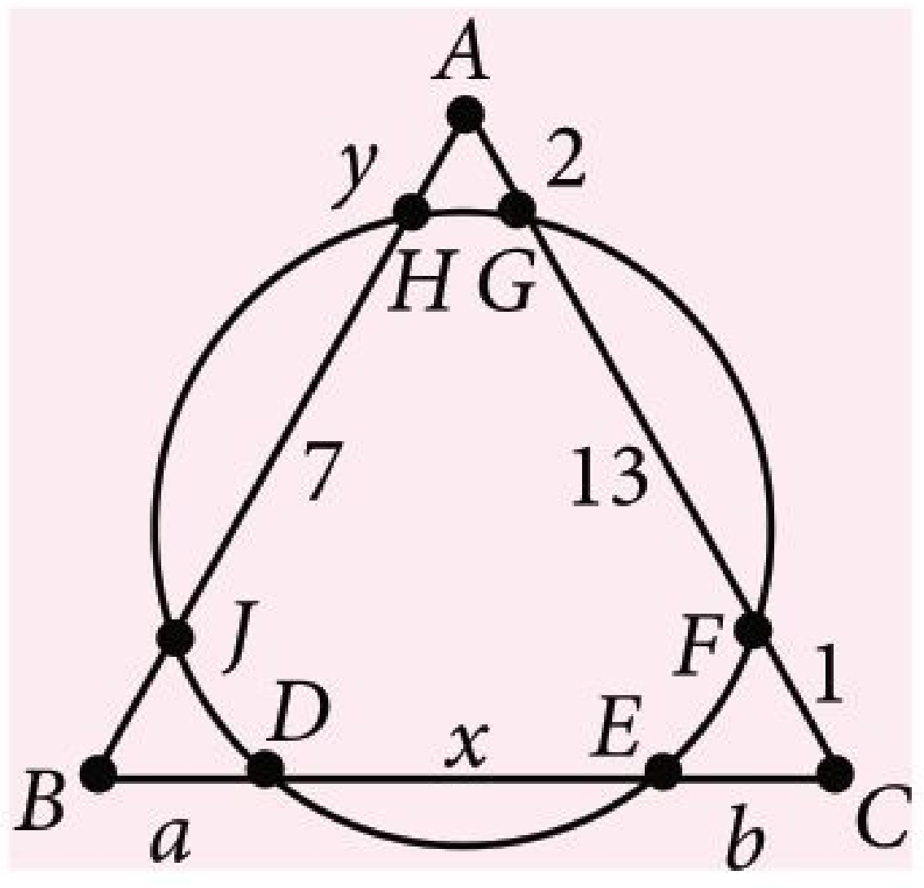
$$f(x) = 2x^2(1 - 3x)$$

On the domain $D = \left\{x \middle| \frac{1}{5} \le x \le \frac{1}{4}\right\}$. In fact, m is the smallest value of f on D, because minimizing f on D is just a restricted version of the original problem: for each $x \in D$, setting $y = 1 - 3x$ and $z = 2x$ gives x, y, z meeting the original constraints, and makes $f(x) = xyz$.



To minimize f on D, first sketch f for all real x. (See Figure.) Since f has a relative minimum at $x = 0$ ($f(x)$ has the same sign as x^2 for $x < \frac{1}{3}$), and cubics have at most one relative minimum, the minimum of f on D must be at one of the end points. In fact,

$$f\left(\frac{1}{4}\right) = \frac{1}{32} \le f\left(\frac{1}{5}\right) = \frac{4}{125}$$



19. (a) : In the adjoining figure, let $AH = y$, $BD = a$, $DE = x$ and $EC = b$. We are given $AG = 2$, $GF = 13$, $HJ = 7$ and $FC = 1$. Thus the length of the side of the equilateral triangle is 16. Also, using the theorem about secants drawn to a circle from an external point, we have $y(y + 7) = 2(2 + 13)$, or

$$0 = y^2 + 7y - 30 = (y - 3)(y + 10)$$

Hence $y = 3$ and $BJ = 6$. Using the same theorem we have $b(b + x) = 1(1 + 13) = 14$ and

$$a(a + x) = 6(6 + 7) = 78.$$

Also, $a + b + x = 16$.

There are many ways of solving the system

(1) $a^2 + ax = 78$,
(2) $b^2 + bx = 14$,
(3) $a + b + x = 16$.

The one given below allows us to find x without first having to find a and b.

Subtract the second equation from the first, factor out $(a - b)$, and use (3):

$$a^2 - b^2 + (a - b)x = (a - b)(a + b + x)$$
$$= (a - b)16 = 78 - 14 = 64.$$

Therefore, $a - b = 4$.

Adding this to (3) we obtain $2a + x = 20$, whence

$$a = 10 - \left(\frac{x}{2}\right)$$

This, substituted into (1), yields

$$\left(10 - \frac{x}{2}\right)\left[10 - \frac{x}{2} + x\right] = \left(10 - \frac{x}{2}\right)\left(10 + \frac{x}{2}\right)$$
$$= 100 - \frac{x^2}{4} = 78$$

$$\frac{x^2}{4} = 22 \quad \Rightarrow \quad x = 2\sqrt{22}$$

PART B

1. Since $b + c > a \Rightarrow a + b + c > 2a$, $1 - a$ is positive; similarly $1 - b$ and $1 - c$ are positive. Therefore

$$(1 - a)(1 - b)(1 - c) > 0$$

i.e., $1 - (a + b + c) + ab + bc + ca - abc > 0$

This yields $-2(ab + bc + ca) + 2abc < -2$

We also have $a^2 + b^2 + c^2 + 2(ab + bc + ca) = 4$

Adding, we get the result.

2. **1st Solution:** Proof by induction : Result is clearly true for $n = 1$. Assume the relation to be true for all $n \le m$. To prove the result for $n = m + 1$

$$\sum_{k=0}^{m+1} \binom{m+1}{k} x^{(k)} y^{(m+1-k)}$$
$$= \sum_{k=0}^{m+1} \left[\binom{m}{k} + \binom{m}{k-1}\right] x^{(k)} y^{(m+1-k)}$$
$$\left(\text{Note : } \binom{m}{m+1} = \binom{m}{-1} = 0\right)$$
$$= \sum_{k=0}^{m} \binom{m}{k} x^{(k)} (y-1)^{(m-k)} y$$
$$+ \sum_{k=0}^{m+1} \binom{m}{k-1} x \cdot (x-1)^{(k-1)} y^{(m+1-k)}$$
$$= y \sum_{k=0}^{m} \binom{m}{k} x^{(k)} (y-1)^{(m-k)} + x \sum_{l=0}^{m} \binom{m}{l} (x-1)^{(l)} y^{(m-l)}$$
$$= y(x + y - 1)^{(m)} + x(x - 1 + y)^{(m)}$$

(by induction hypothesis)

$$= (x + y)(x + y - 1)^{(m)}$$
$$= (x + y)^{(m - 1)}$$

2nd Solution: By using Leibnitz's theorem for successive differentiation. If u and v are two functions of a variable t, each being differentiable n times then uv also has n^{th} derivative given as follows :

$$\frac{d^n(uv)}{dt^n} = \sum_{k=0}^{n} \binom{n}{k} \frac{d^k u}{dt^k} \cdot \frac{d^{n-k} v}{dt^{n-k}}.$$

Let $u = t^x$, $v = t^y$. Then L.H.S. is

$$(x + y)(x + y - 1) \ldots (x + y - n + 1)t^{x + y - n}$$

$$(x + y)^{(n)}\ t^{x + y - n}$$

R.H.S. is

$$\sum_{k=0}^{n} \binom{n}{k} x(x-1)\ldots(x-k+1)\, t^{x-k} \times y(y-1)$$

$$\ldots.\ (y - (n - k) + 1)t^{y - (n - k)}$$

$$= \left(\sum_{k=0}^{n} \binom{n}{k} x^{(k)} y^{(n-k)} \right) t^{x+y-n}$$

So cancelling $t^{x + y - n}$ we get the required result.

3. Let $\alpha = x + y$ and $\beta = xy$. Then

$$x^2 + y^2 = \alpha^2 - 2\beta \text{ and } x^3 + y^3 = \alpha^3 - 3\alpha\beta$$

Therefore the given equations can be written as

$$\alpha^3 - 3\alpha\beta = 7 \text{ and } \alpha^3 + \alpha - \beta = 4$$

Also note that α is positive, since

$$\frac{1}{2}(x+y)[(x-y)^2 + x^2 + y^2] = x^2 + y^2 = 7$$

Now $\beta = \alpha^2 + \alpha - 4$ and $\alpha^3 - 3\alpha(\alpha^2 + \alpha - 4) = 7$

$\Rightarrow\quad 2\alpha^3 + 3\alpha^2 - 12\alpha + 7 = 0$

$\Rightarrow\quad (\alpha - 1)^2 (2\alpha + 7) = 0$

$\Rightarrow\quad \alpha = 1\quad (\because\ \alpha > 0)$

So, $\beta = \alpha^2 + \alpha - 4 = -2$

i.e., $x + y = 1$ and $xy = -2$.

Solving we get $(x, y) = (2, -1)$ or $(-1, 2)$.

4. For any $m \in Z$, let $\theta(m)$ denote the highest power of 3 which divides m. Let $t_n = 2^{3^n} + 1$ where $n \in N$. Then for any $n \in N$,

$$t_{n+1}(2^{3^n})^3 + 1 = (t_n - 1)^3 + 1$$

Which by simplification gives $t_{n+1} = t_n^3 - 3t_n^2 + 3t_n$. Now let us show by induction $\theta(t_n) = 3^{n+1}$. When $n = 1$, it is obvious. Assume $n \geq 1$ such that $\theta(t_n) = 3^{n+1}$. Then

$$\theta(t_{n+1}) = \theta(t_n^3 - 3t_n^2 + 3t_n)$$

$$= \theta(t_n)\ \theta\ (t_n^2 - 3t_n + 3) = 3^{n+1} \cdot 3 = 3^{n+2}$$

5. Consider the product $n(n + 1)(n + 2)(n + 3)$ of four consecutive numbers. Suppose $n > 1$ as $1 \cdot 2 \cdot 3 \cdot 4 = 24$ is anyway not a cube. We use the fact that if the product of two relatively prime numbers is a cube then each of the two numbers is itself a cube.

Case-I: Suppose n is even. Then $(n + 1)$ is relatively prime to $n(n + 2)(n + 3)$. Thus if $n(n + 1)(n + 2)(n + 3)$ is a cube we must have that $(n + 1)$ and $n(n + 1)(n + 3)$ are cubes. But $n(n + 2)(n + 3)$ is not a cube since it lies strictly between two consecutive cubes:

$$(n + 1)^3 = n^3 + 3n^2 + 3n + 1 < n(n + 2)(n + 3)$$

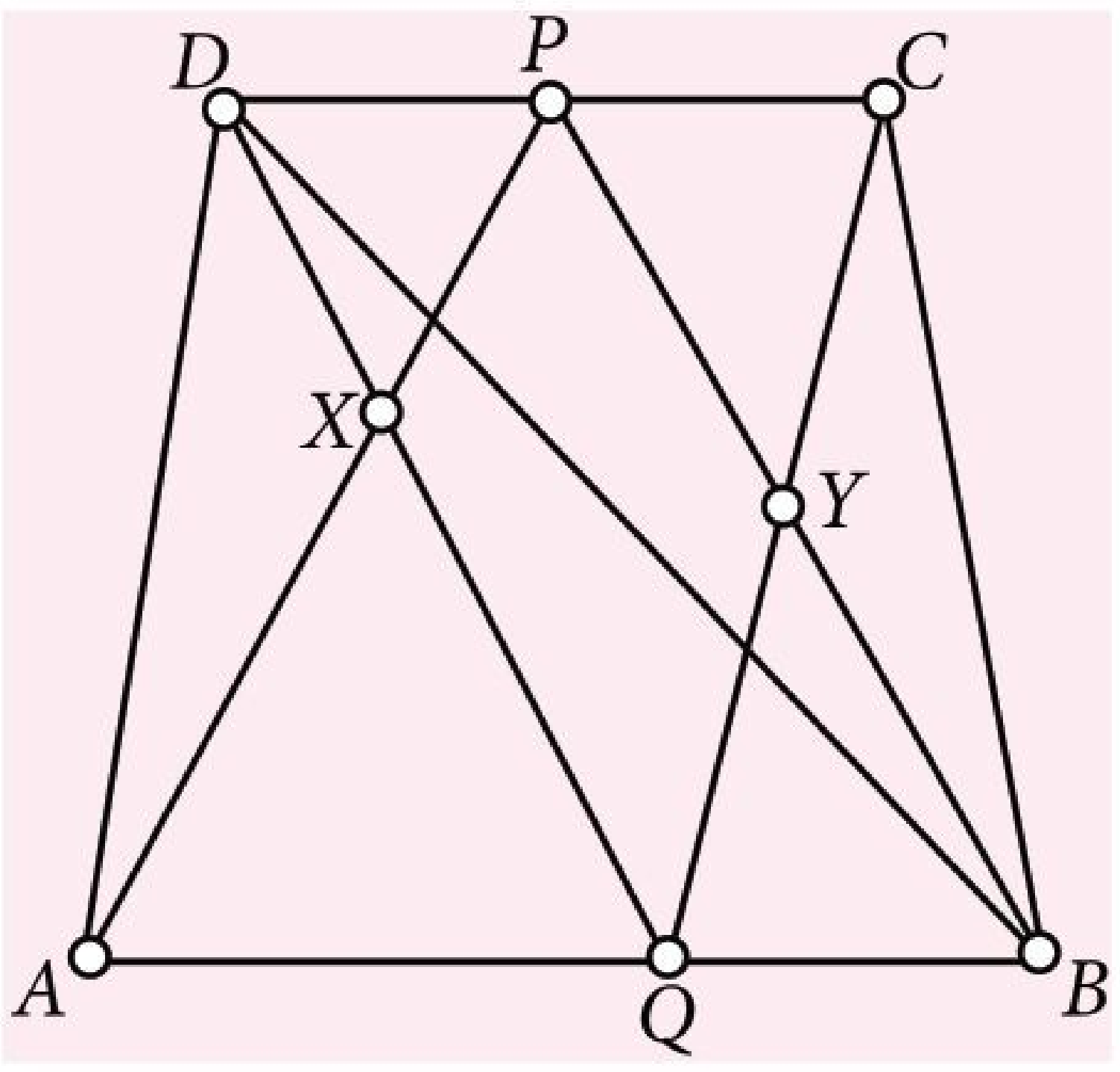
and $n(n + 2)(n + 3) < (n + 2)^3 = n^3 + 6n^2 + 12n + 8$

Thus, $n(n + 1)(n + 2)(n + 3)$ is not a cube for even n.

Case-II: Suppose n is odd. The proof is similar by noting that this time

$n(n + 1)(n + 3)$ is prime to $(n + 2)$.

6. We denote areas of triangles ABC, quadrilaterals $ABCD$, etc. by $[ABC]$, $[ABCD]$ etc. Join PQ and draw one of the diagonals, say BD. We use the fact that the median of a triangle bisects its area.



From triangles DAB (with median DQ) and BCD (with median BP), we have

$$[ADQ] = [BDQ] \text{ and } [BPC] = [BPD].$$

Adding, we have

$$[ADQ] + [BPC] = [BDQ] + [BPD]$$

$$= [BPDQ] = [BPQ] + [DPQ]$$

$$= [APQ] + [CPQ],$$

Since PQ is a median of both the triangles APB and CQD. Writing in terms of smaller areas, we have

$$[AXQ] + [AXD] + [BYC] + [PYC]$$

$$= [AXQ] + [PXQ] + [CPY] + [QPY].$$

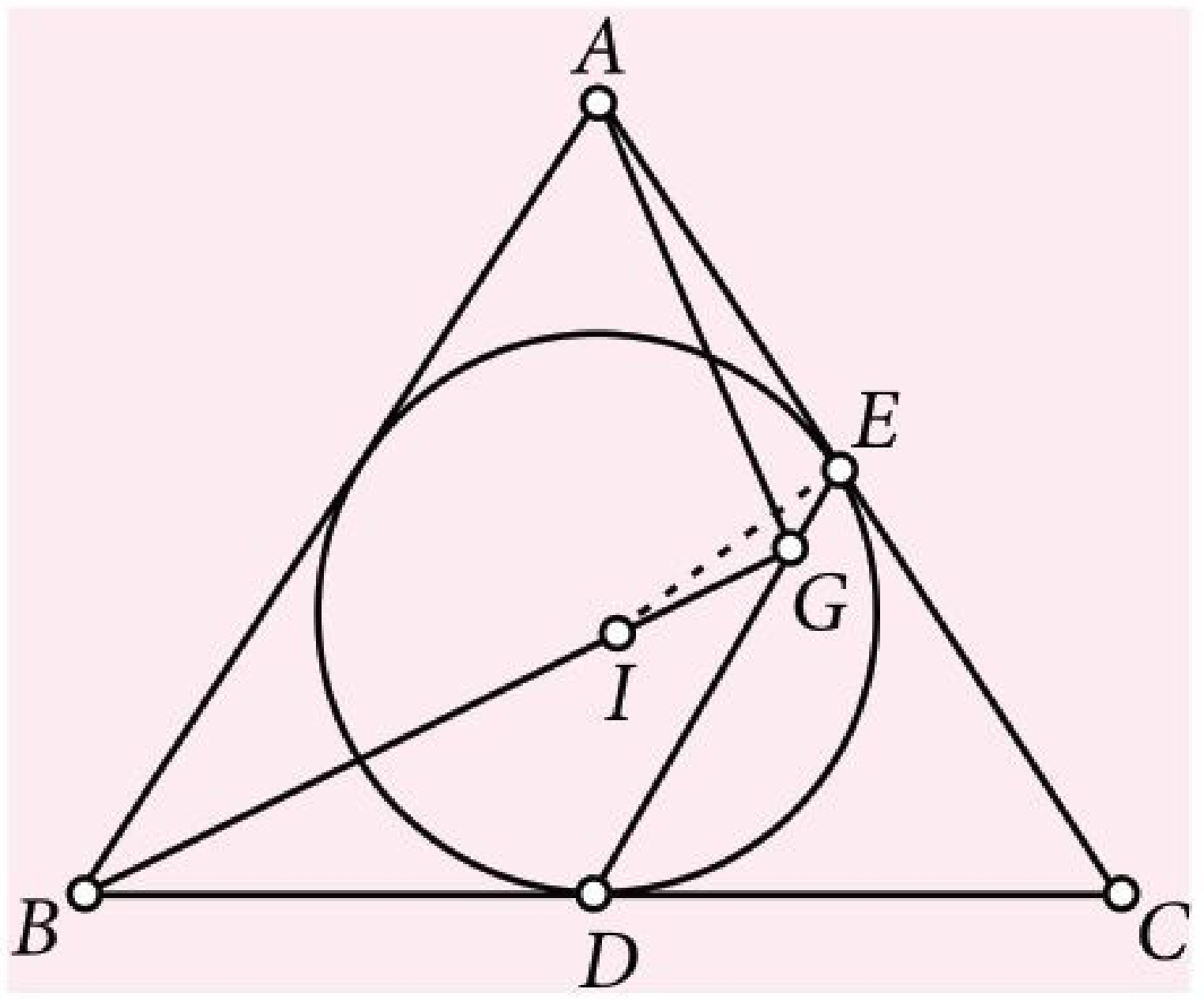
On cancellation, this yields,

$$[ADX] + [BCY] = [PXQY].$$

If $ABCD$ is a concave quadrilateral and the points P, Q, X, Y are located as in the problem, then by a similar argument, we arrive at the relation $|[ADX] - [BCY]| = [PXQY]$, where the left hand side denotes the modulus of the difference of areas.

7. **1st Solution:** Join ID and AI. We first show that triangles BDG and BIA are similar. We have

$$\angle DBG = \frac{B}{2} = \angle IBA \qquad \ldots.(1)$$



Further ΔCDE is isosceles, because $CD = CE$. So

$$\angle CDE = \angle CED = 90^\circ - \frac{C}{2}.$$

Therefore $\angle BDG = 180^\circ - \angle CDE = 90^\circ + \dfrac{C}{2}.$

Also, $\angle BIA = 180^\circ - \angle ABI - \angle BAI$

$$= 180^\circ - \frac{B}{2} - \frac{A}{2} = 90^\circ + \frac{C}{2}$$

Thus, $\angle BDG = \angle BIA$ ....(2)

From (1) and (2), it follows that, ΔBDG is similar to ΔBIA. Therefore $\dfrac{BD}{BI} = \dfrac{BG}{BA}.$

This fact along with the relation $\angle DBI = \angle GBA\left(=\dfrac{B}{2}\right)$ implies that triangles DBI and GBA are similar. Consequently, $\angle BDI = \angle BGA$. But $\angle BDI = 90^\circ$. So $\angle BGA = 90^\circ$. That is, BG is perpendicular to AG.

2nd Solution: Join IE. We show that $IGEA$ is a cyclic quadrilateral. As in the previous method,

$$\angle CED = \angle CDE = 90^\circ - \frac{C}{2}.$$

Therefore, $\angle AEG = 180^\circ - \angle DEC = 180^\circ - \left(90^\circ - \dfrac{C}{2}\right)$

$$= 90^\circ + \frac{C}{2}$$

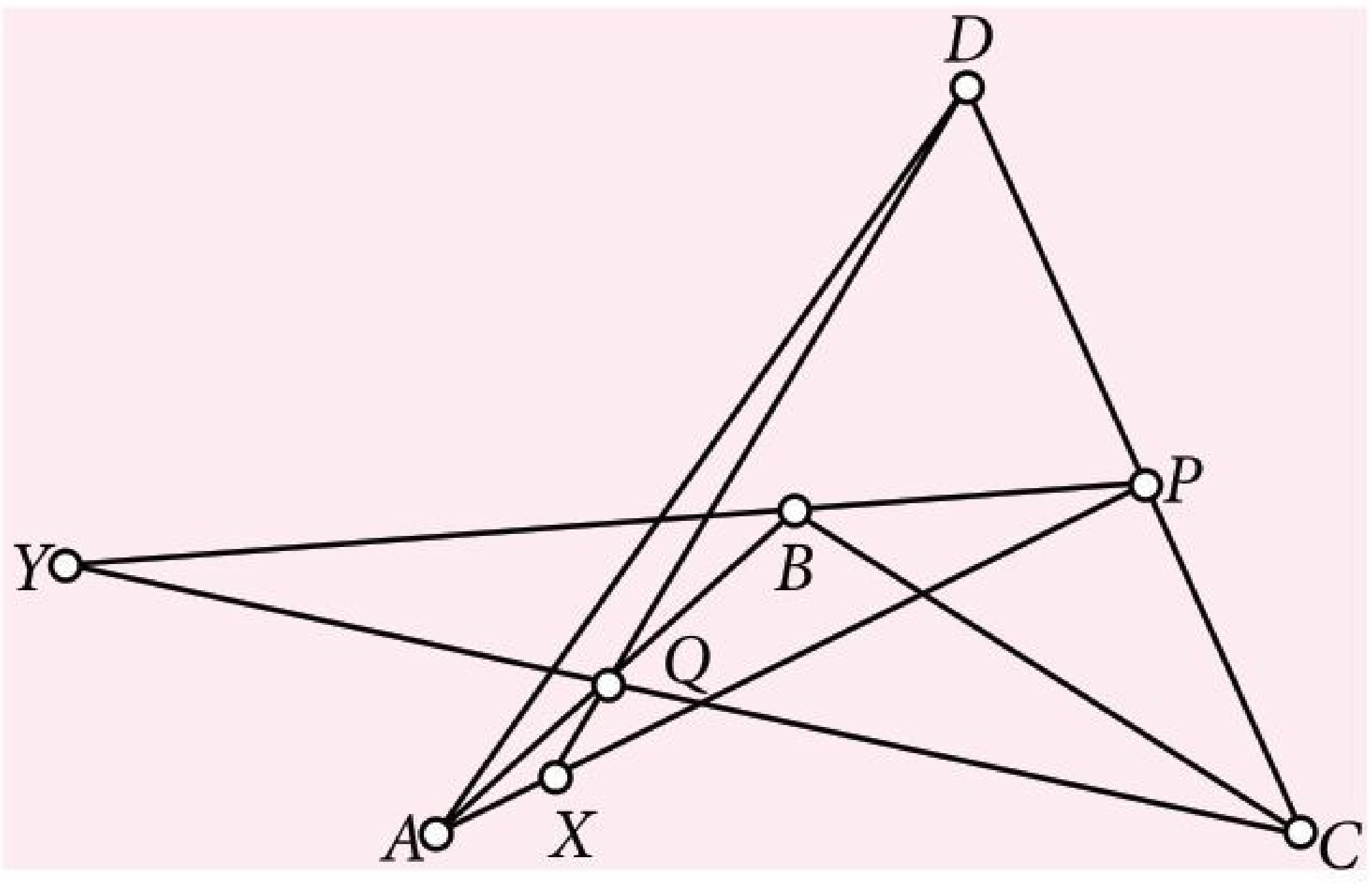
Also, $\angle AIG = \angle ABI + \angle BAI = \dfrac{B}{2} + \dfrac{A}{2}$

So, $\angle AEG + \angle AIG = 90^\circ + \dfrac{C}{2} + \dfrac{B}{2} + \dfrac{A}{2} = 180^\circ$

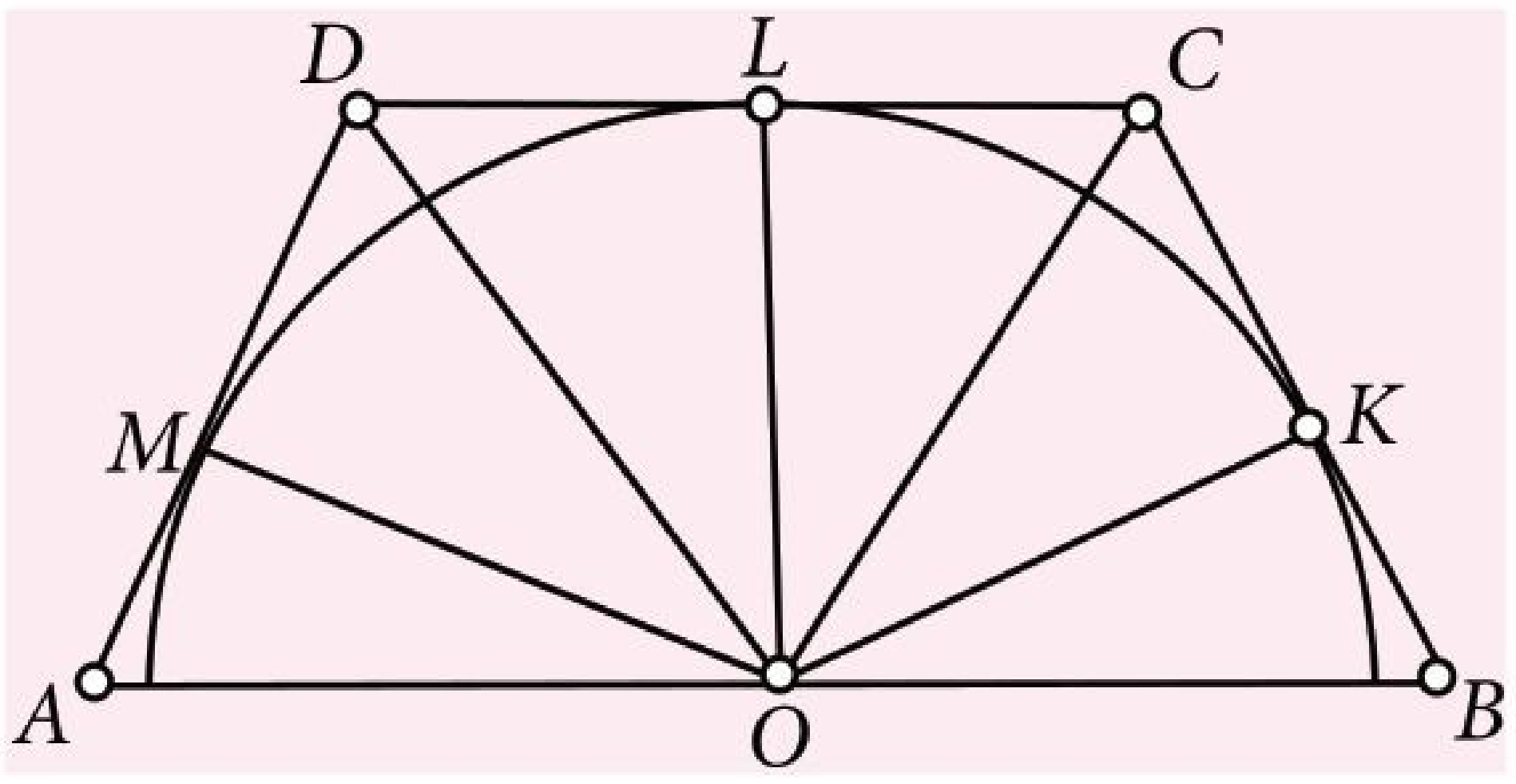
This implies that $IGEA$ is a cyclic quadrilateral. So

$$\angle AGI = \angle AEI = 90^\circ.$$

That is AG is perpedicular to GI, as required.



8. Let K, L, M be the points of contact of the semicircle with the sides BC, CD, DA respectively. Join OK, OL, OM, OC and OD where O is the centre of the circle as well as the midpoint of AB.



In the right angled triangles AOM and BOK, we have $AO = BO$ and $OM = OK$. Hence these two triangles are congruent. Thus

$$\angle AOM = \angle BOK = \alpha,\ \angle DOM = \angle DOL = \beta \text{ and } \angle COL = \angle COK = \gamma.$$

Adding up the angles formed at O, we obtain $2\alpha + 2\beta + 2\gamma = 180^\circ$ and so $\alpha + \beta + \gamma = 90^\circ$.

In triangles AOD and BCO, we have

$$\angle OAD = 90^\circ - \angle AOM = 90^\circ - \alpha\ ;$$

and $\angle CBO = 90^\circ - \angle KOB = 90^\circ - \alpha$.

Similarly $\angle AOD = \alpha + \beta = 90^\circ - \gamma$;

and $\angle BCO = 90^\circ - \angle COK = 90^\circ - \gamma$.

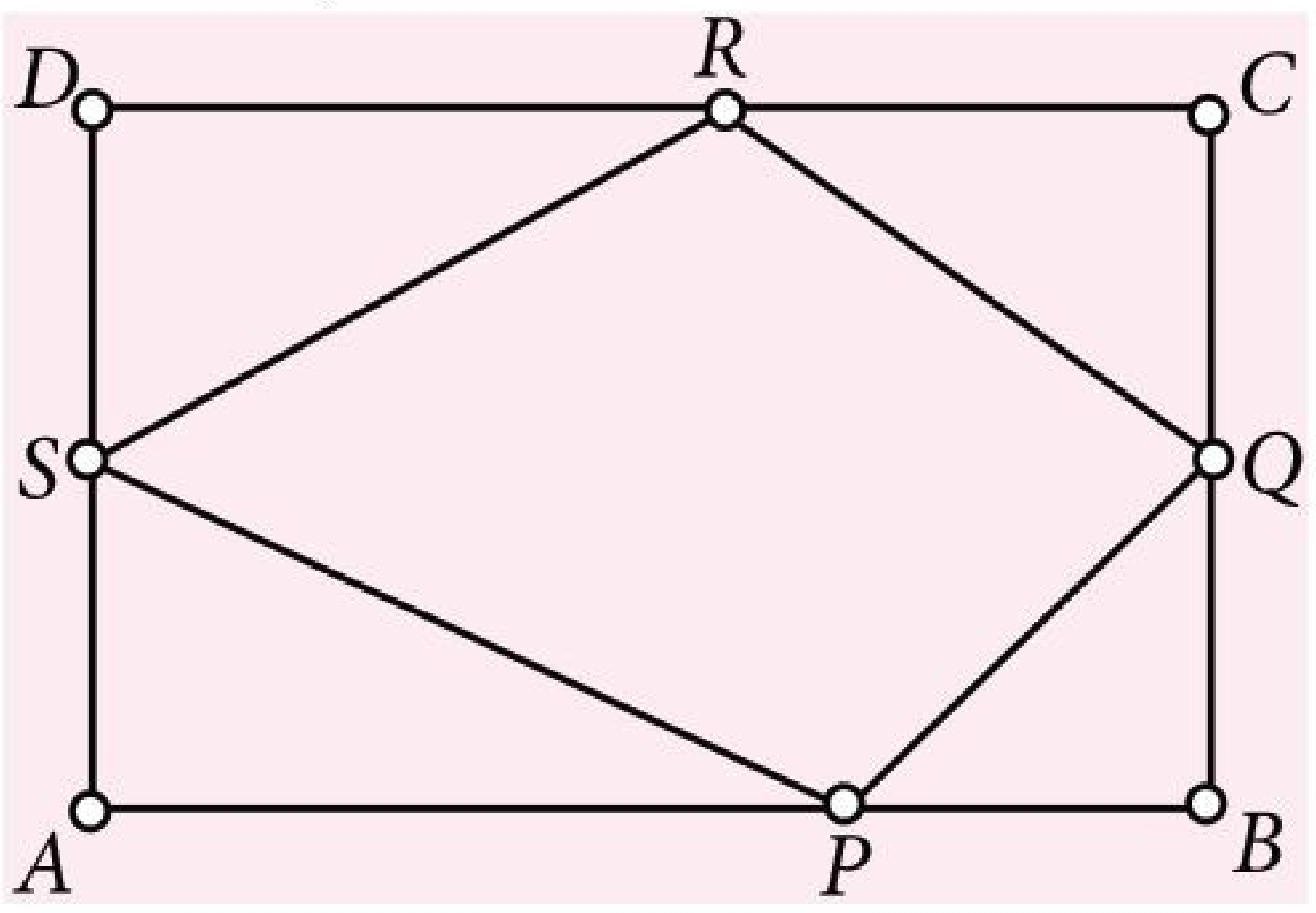
Therefore, triangle AOD is similar to triangle BCO.

Consequently, $\dfrac{AO}{AD} = \dfrac{BC}{BO}.$

So, $AD \cdot BC = AO \cdot BO = \dfrac{1}{4}AB^2$

That is $AB^2 = 4AD \cdot BC$.

9. We have (see figure) $PQ \cdot QR > BQ \cdot QC$, $QR \cdot RS > CR \cdot RD$, etc.



Therefore,

$$\begin{aligned}(PQ + QR + RS + SP)^2 &= PQ^2 + \dots + 2PQ \cdot QR + \dots\\ &> (PB^2 + BQ^2) + \dots + 2BQ \cdot QC + \dots\\ &= (PA + PB)^2 + (BQ + QC)^2 + (CR + RD)^2 + (DS + SA)^2\\ &= AB^2 + BC^2 + CD^2 + DA^2\\ &= AC^2 + BD^2 = 2AC^2\end{aligned}$$

Hence $PQ + QR + RS + SP > \sqrt{2}\ AC$.

10. Let us start counting 3-term G.P.'s with common ratios $2, 2^2, 2^3, \dots$

The 3-term G.P.'s with common ratio 2 are

$$1, 2, 2^2; 2, 2^2, 2^3; \ldots; 2^{n-2}, 2^{n-1}, 2^n.$$

They are $(n - 1)$ in number. The 3-term GP's with common ratio 2^2 are

$$1, 2^2, 2^4; 2, 2^3, 2^5; \ldots; 2^{n-4}, 2^{n-2}, 2^n.$$

They are $(n - 3)$ in number. Similarly we see that the 3-term GP's with common ratio 2^3 are $(n - 5)$ in number and so on. Thus the number of 3-term GP's which can be formed from the sequence $1, 2, 2^2, 2^3, \ldots, 2^n$ is equal to $S = (n - 1) + (n - 3) + (n - 5) + \ldots$

Here the last term is 2 or 1 according as n is odd or even. If n is odd, then

$$S = (n - 1) + (n - 3) + (n - 5) + \ldots + 2$$

$$= 2\left(1+2+3+\ldots+\frac{n-1}{2}\right) = \frac{n^2-1}{4}$$

If n is even, then

$$S = (n-1) + (n-3) + \ldots + 1 = \frac{n^2}{4}$$

Here the required number is $\frac{(n^2-1)}{4}$ or $\frac{n^2}{4}$ according as n is odd or even.

11. (a) Let a, b, c be the sides of a triangle with $a + b + c = 1996$, and each being a positive integer. Then $a + 1$, $b + 1$, $c + 1$ are also sides of a triangle with perimeter 1999 because

$$a < b + c \quad \Rightarrow \quad a + 1 < (b + 1) + (c + 1),$$

and so on. Moreover (999, 999, 1) form the sides of a triangle with perimeter 1999, which is not obtainable in the form $(a + 1, b + 1, c + 1)$ where a, b, c are the integers and the sides of a triangle with $a + b + c = 1996$. We conclude that $f(1999) > f(1996)$.

(b) As in the case (a) we conclude that $f(2000) \geq f(1997)$. On the other hand, if x, y, z are the integer sides of a triangle with $x + y + z = 2000$, and say $x \geq y \geq z \geq 1$, then we cannot have $z = 1$; for otherwise we would get $x + y = 1999$ forcing x, y to have opposite parity so that $x - y \geq 1 = z$ violating triangle inequality for x, y, z. Hence $x \geq y \geq z \geq 1$. This implies that $x - 1 \geq y - 1 \geq z - 1 > 0$. We already have $x < y + z$. If $x \geq y + z - 1$, then we see that $y + z - 1 \leq x < y + z$, showing that $y + z - 1 = x$. Hence we obtain $2000 = x + y + z = 2x + 1$ which is impossible.

We conclude that $x < y + z - 1$. This shows that $x - 1 < (y - 1) + (z - 1)$ and hence $x - 1$, $y - 1$, $z - 1$ are the sides of a triangle with perimeter 1997. This gives $f(2000) \leq f(1997)$. Thus we obtain the desired result.

◆ ◆

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MATHEMATICS CLASSROOM STUDY PACKAGE

CONCEPT
MAP

SETS

Class XI

Representation of Sets

- **Set Builder Form :** A set is described by characterising property $P(x)$ of its element x.
- **Roster or Tabular Form :** A set is described by listing all the elements separated by commas.

Cardinal Properties

If A, B and C are finite sets, U be the finite universal set, then

- $n(A - B) = n(A) - n(A \cap B)$
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B) = n(A) + n(B)$, if A, B are disjoint.
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
- $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$
- $n(\text{exactly two}) = n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
- $n(\text{exactly one}) = n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + 3n(A \cap B \cap C)$

SETS

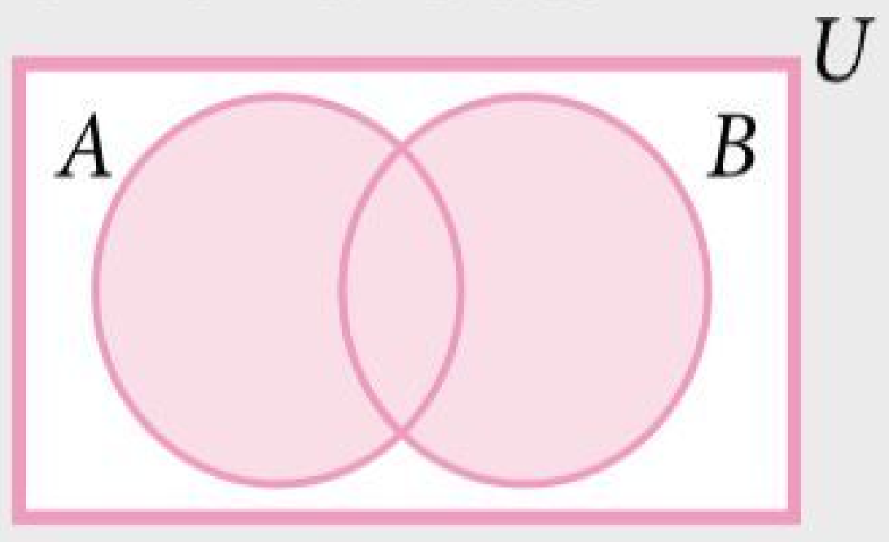
Collection of well defined objects

Types of Sets

- **Singleton set :** A set with only one element.
- **Empty set :** A set with no element.
- **Finite set :** A set contains finite number of elements.
- **Infinite set :** Set which is not finite.
- **Equal sets :** Two sets A and B are equal if they have exactly same elements.
- **Equivalent sets :** Two finite sets A and B are equivalent if they have same number of elements.
- **Universal set :** The set that contains all the sets in the given context.
- **Subset :** A set A is said to be subset of set B if each element of set A is also an element of set B *i.e.*, $A \subseteq B$.
- **Proper subset :** If set A is a subset of set B but $A \neq B$, then $A \subset B$.
- **Superset :** If set A is a subset of set B *i.e.*, $A \subseteq B$, then set B is called superset.
- **Power set :** Set of all subsets of a given set.

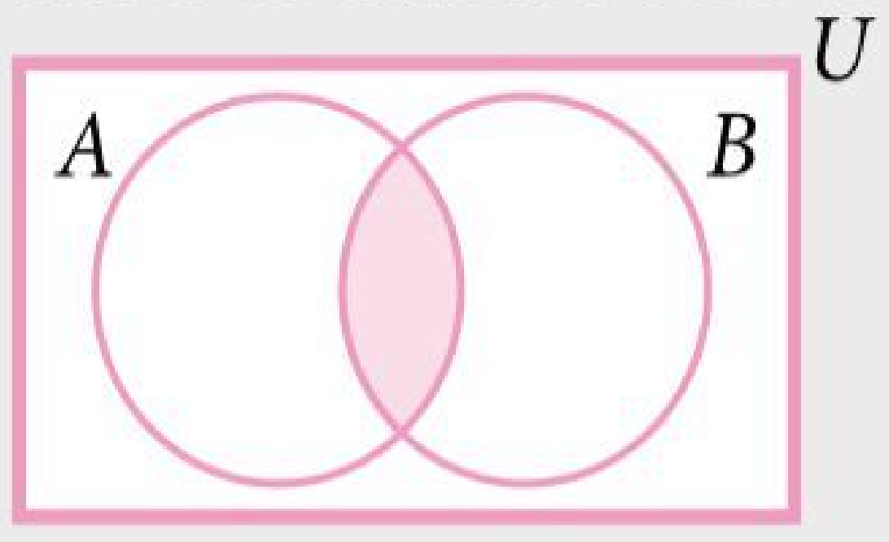
Venn Diagrams

- **Union of Sets**



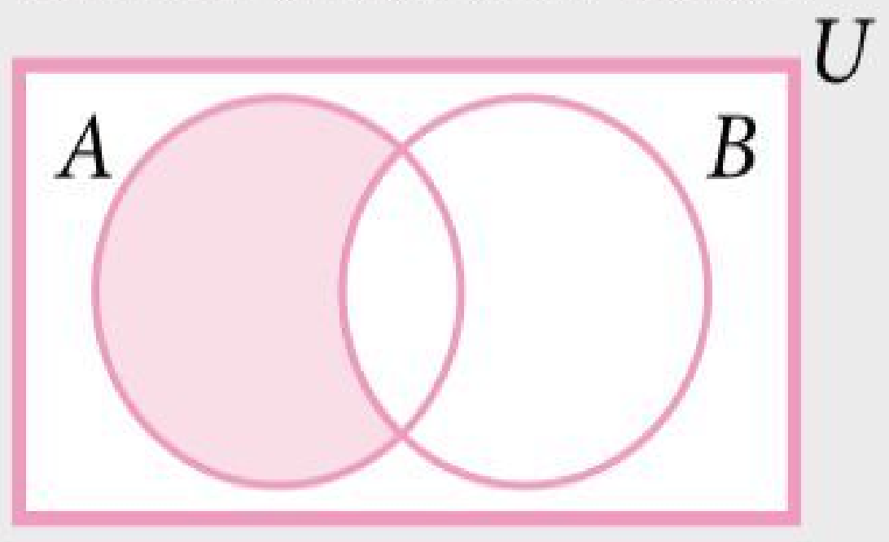
Shaded part represents $A \cup B$

- **Intersection of Sets**



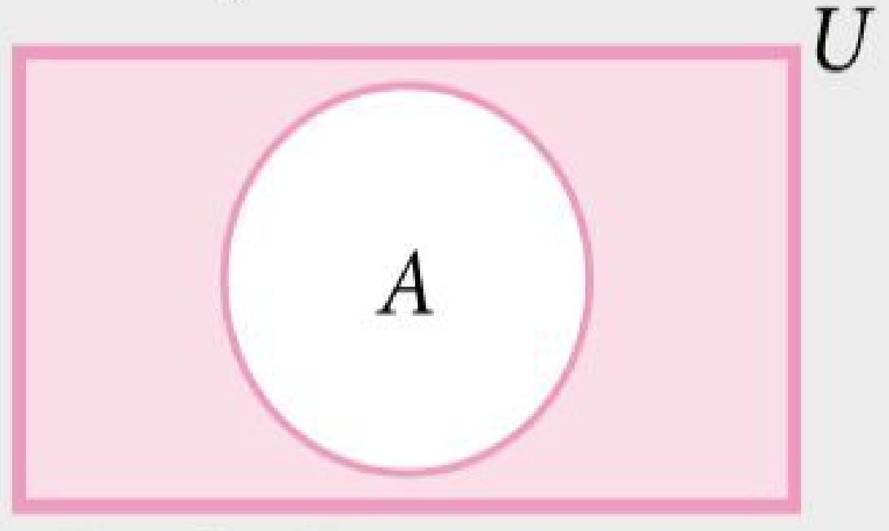
Shaded part represents $A \cap B$

- **Difference of Sets**



Shaded part represents $A - B$

- **Complement of Sets**



Shaded part represents A'

Operations on Sets

Laws of Algebra of Sets

- **Idempotent :** $A \cup A = A$; $A \cap A = A$
- **Commutative :** $A \cup B = B \cup A$; $A \cap B = B \cap A$
- **Associative :** $A \cup (B \cup C) = (A \cup B) \cup C$
  $A \cap (B \cap C) = (A \cap B) \cap C$
- **Identity :** $A \cup \phi = \phi \cup A = A$, $A \cap \phi = \phi$
  $A \cap U = U \cap A = A$, $A \cup U = U$
- **Distributive :**
  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

- **Complement Law :** $A \cup A' = U$; $A \cap A' = \phi$
- **De Morgan's Law :** $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$
- **Law of Double Complement :** $(A')' = A$
- **Law of Empty Set and Universal Set :** $\phi' = U$ and $U' = \phi$

Union : It consists of all the elements of set A and set B with common element taken once *i.e.*, $(A \cup B) = \{x : x \in A \text{ or } x \in B\}$

Intersection : It consists of those elements which belong to both set A and B *i.e.*, $(A \cap B) = \{x : x \in A \text{ and } x \in B\}$

Complement : Set of all elements of universal set U which are not the elements of set A *i.e.*, $A' = U - A$.

Difference : Set of elements which belong to set A but not to set B *i.e.*, $A - B = \{x : x \in A, x \notin B\}$

APPLICATION OF DERIVATIVES

CONCEPT MAP

Class XII

Increasing and Decreasing Functions

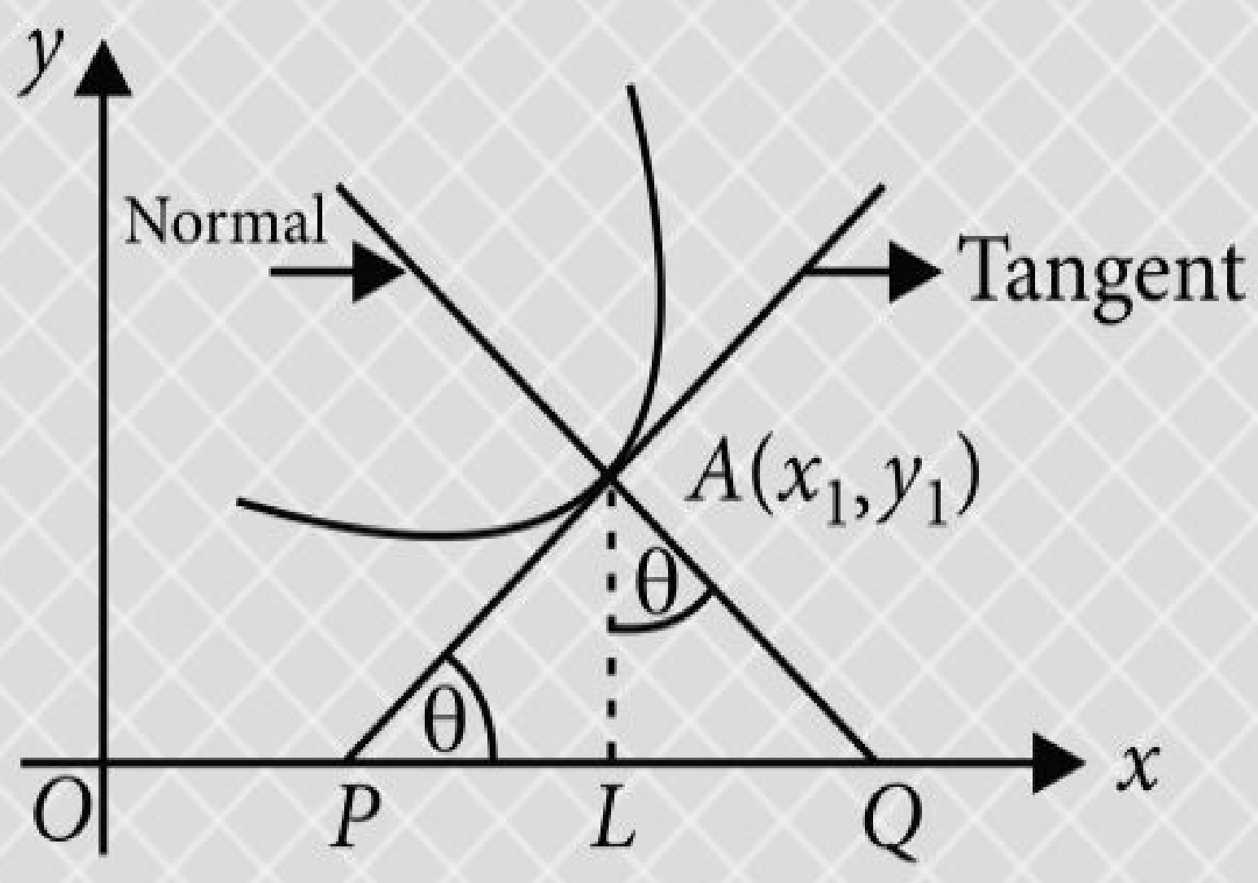
- A function f is said to be an increasing function in (a, b), if $x_1 < x_2$ $\Rightarrow f(x_1) < f(x_2)\ \forall\ x_1, x_2 \in (a, b)$.
- A function f is said to be a decreasing function in (a, b), if $x_1 < x_2$ $\Rightarrow f(x_1) > f(x_2)\ \forall\ x_1, x_2 \in (a, b)$.
- $f(x)$ is known as non-decreasing if $f'(x) \geq 0$ and non-increasing if $f'(x) \leq 0$.
- A function f is said to be monotonic in an interval if it is either increasing or decreasing in that interval.

Tangents and Normals

- Equation of tangent to the curve $y = f(x)$ at point $P(x_1, y_1)$ is $y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1,\, y_1)}(x - x_1)$
- Equation of normal to the curve $y = f(x)$ at point $P(x_1, y_1)$ is $y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,\, y_1)}}(x - x_1)$

Length of Tangents and Normals

- Length of tangent $= AP$ $= \left|\frac{y_1\sqrt{1+\left(\frac{dy_1}{dx_1}\right)^2}}{\frac{dy_1}{dx_1}}\right|$



- Length of normal $= AQ = \left|y_1\sqrt{1+\left(\frac{dy_1}{dx_1}\right)^2}\right|$
- Length of sub-tangent $= \left|\frac{y_1}{\frac{dy_1}{dx_1}}\right|$
- Length of sub-normal $= \left|y_1\frac{dy_1}{dx_1}\right|$

Rate of Change of Quantities

- Let $y = f(x)$, then $\frac{dy}{dx}$ or $f'(x)$ denotes the rate of change of y w.r.t. x.

Errors and Approximations

Let $y = f(x)$, Δx be the small change in x and Δy be the corresponding change in y. Then, $\Delta y = \frac{dy}{dx}(\Delta x)$

- **Absolute Error :** Δx
- **Relative Error :** $\Delta x/x$
- **Percentage Error :** $\left(\frac{\Delta x}{x}\times 100\right)$

Maxima and Minima

- **Local Maxima :** At $x = a$; $f(a - h) < f(a) > f(a + h)\ \forall\ h \to 0^+$
- **Local Minima :** At $x = a$; $f(a - h) > f(a) < f(a + h)\ \forall\ h \to 0^+$
- **Extremum :** At $x = a$; $f(x)$ is said to have extremum if it has either maximum or minimum.

First Derivative Test

- $x = a$ be the point of local maxima if $f'(a) = 0$ and $f'(x)$ changes its sign from +ve to –ve.
- $x = a$ be the point of local minima if $f'(a) = 0$ and $f'(x)$ changes its sign from –ve to +ve.

Second Derivative Test

- Find the roots of $f'(x) = 0$.
- Suppose $x = a$ is one of the roots.
- At $x = a$, if $f''(x) < 0 \Rightarrow f(x)$ will attain local maximum value, if $f''(x) > 0 \Rightarrow f(x)$ will attain local minimum value.

Higher Order Derivative Test

- If $f'(a) = f''(a) = \ldots\ldots = f^{n-1}(a) = 0$ and $f^{n}(a)$ is a finite number other than zero, then
    - If n is even and $f^{n}(a) < 0 \Rightarrow x = a$ is a point of local maxima.
    - If n is even and $f^{n}(a) > 0 \Rightarrow x = a$ is a point of local minima.
    - If n is odd $\Rightarrow x = a$ is neither a point of maxima nor a point of minima.

beat the TIME TRAP

Duration : 30 minutes

SECTION-I

Single Option Correct Type

1. If $A=\sum_{r=0}^{\infty}\left\{\left(-\frac{1}{2}\right)^r \sin^{2r}x\right\}, B=\sum_{r=0}^{\infty}\sin^{2r}x,$ then number of solution in $[-2\pi, 2\pi]$ of $A:B = 4\sin^2 x : (1+\cos 2x)$ is

(a) 2 (b) 4 (c) 8 (d) none of these

2. $\lim_{x\to\infty}\left[\dfrac{\sum_{r=1}^{2013}(r+x)^{2013}}{x^{2013}+2013^{2013}}\right]=$

(a) 2014 (b) 2013 (c) $\frac{1}{2013}$ (d) none of these

3. If $\int\frac{dx}{\sqrt[3]{x^2}-\sqrt[3]{x}}=\alpha\left[\sqrt[\gamma]{x}+\log_e|\sqrt[\gamma]{x}-1|\right]+c$, then

(a) α, β, γ are in A.P.
(b) α, β, γ are in G.P.
(c) $\alpha=\beta=\gamma$ (d) all of these

4. If $f(x)$ be an identity function, then equation $\sum_{r=1}^{3}\{f(x)-(2012+r)\}^{-1}=0$ has

(a) no real roots (b) real and equal roots
(c) real and different roots
(d) none of these

5. If $\sin x : \sin y : \sin z = \cos A : \cos B : \cos C$ then $\sum\left(\frac{\sin^2 A-\cos^2 x}{\sin x-\cos A}\right)=$

(a) $\frac{\sum\sin^2 x-\sum\cos^2 A}{(\sum\sin x)(\sum\cos A)}$ (b) $\frac{\sum\sin A\cos x}{\sum\sin x-\sum\cos A}$

(c) $\frac{\sum(\sin A-\cos x)^2}{\sum\sin A\cos x}$

(d) $\frac{(\sum\sin x)^2-(\sum\cos A)^2}{\sum(\sin x-\cos A)}$

6. If $\sum_{r=0}^{\infty}\left(x^r+x^{r+1}+x^{r+2}\right)=\sum_{r=0}^{\infty}p_r x^r\ (|x|<1)$, then $\sum_{r=0}^{671}p_r=$

(a) 0 (b) 2012 (c) 2015 (d) 2013

7. If $\int(1+x\tan x)^{-2}dx=\frac{1}{x+f(x)}+c$, then $f(x)$ =

(a) $x\tan x$ (b) $\cot x$
(c) $\tan x$ (d) none of these

8. If $x\sin^2\alpha + y + z = 0$, $x + y\sin^2\beta + z = 0$ & $x + y + z\sin^2\gamma = 0$ ($\alpha \neq \beta \neq \gamma \neq (2n+1)\pi/2$, where $n \in I$) have a non-trivial solution, then $\sum\tan^2\alpha$ =

(a) 0 (b) 1 (c) 2 (d) none of these

9. If $a_i > 0$ $(i = 1, 2, 3, 4)$ so that $502a_1 + 503a_2 + 504a_3 + 505a_4 = 2014$ and $256\,a_1a_2a_3a_4 \geq \left(\sum_{r=1}^{4}a_r\right)^4$, then $\sum_{r=1}^{4}a_r^r=$

(a) 2014 (b) 1 (c) 4 (d) none of these

10. If x and y be two real variables satisfying $x^2+y^2=\frac{t^2-1}{t}$ and $x^4+y^4=\frac{t^4+1}{t^2}$, then which of the following is(are) true?

(a) $y^2 + x^{-2} = 0$ (b) $x^3 y=\frac{dx}{dy}$

(c) $xdy + ydx = 0$ (d) none of these

SECTION-II

Multiple Options Correct Type

11. If $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}\perp\vec{b}, c$ is inclined at the same angle to both $\vec{a}$ and $\vec{b}$ and $\vec{c}=p\vec{a}+q\vec{b}+r(\vec{a}\times\vec{b})$, then

(a) $p = q$ (b) $|p| \le 1$

(c) $|q| \le 1$ (d) $pq > 1$

12. If a, b, c be three positive numbers and $(ab + bc + ca)x^2 + (a + b + c)x + 1 = 0$ has complex roots, then

(a) $\sqrt{a}+\sqrt{c}>\sqrt{b}$

(b) $\sqrt{a}+\sqrt{b}+\sqrt{c}>4$

(c) $\left(\frac{1}{\sqrt{ab}}+\frac{1}{\sqrt{bc}}-\frac{1}{\sqrt{ca}}\right)\left(\frac{1}{\sqrt{bc}}+\frac{1}{\sqrt{ca}}-\frac{1}{\sqrt{ab}}\right)\times\left(\frac{1}{\sqrt{ca}}+\frac{1}{\sqrt{ab}}-\frac{1}{\sqrt{bc}}\right)>0$

(d) none of these

13. If $a\sin\theta + b\cos\theta = c = a\operatorname{cosec}\theta + b\sec\theta$, then

(a) $\sin 2\theta=\dfrac{2ab}{c^2-a^2-b^2}$

(b) $\tan^3\theta=\dfrac{a}{b}$

(c) $a\cos^3\theta + b\sin^3\theta = 0$

(d) $\sin\theta+\operatorname{cosec}\theta=\dfrac{a^2+c^2-b^2}{ac}$

14. Let $f(x)=x^{\frac{1}{\{x\}}}$ (where $\{\cdot\}$ denotes the fractional part of x), then

(a) $\lim\limits_{x\to 0^+} f(2013^x)=2013$

(b) $\lim\limits_{x\to 0^-} f(2013^x)=1$

(c) $\lim\limits_{x\to 0^+} f(2013^x)=e$

(d) $\lim\limits_{x\to 0} f(2013^x)$ does not exist

15. A person draws 3 balls randomly from a bag containing 3 white and 3 black balls and then he put 3 red balls into the bag and draws 3 balls again randomly. The probability that now he has all 3 balls of different colour is

(a) > 20 % (b) > 25 %

(c) > 30 % (d) > 33 %

SOLUTIONS

1. (c) :

$$A=\sum_{r=0}^{\infty}\left\{\left(-\frac{1}{2}\right)^r\sin^{2r}x\right\}=1-\frac{1}{2}\sin^2x+\frac{1}{4}\sin^4x-\dots\text{ to}\infty$$

$$=\frac{1}{1-\left(-\frac{1}{2}\sin^2x\right)}=\frac{2}{2+\sin^2x}$$

$$B=\sum_{r=0}^{\infty}\sin^{2r}x=1+\sin^2x+\sin^4x+\dots\text{to}\infty=\frac{1}{\cos^2x}$$

$$\therefore\ A:B=4\sin^2x:(1+\cos 2x)\Rightarrow\frac{2\cos^2x}{2+\sin^2x}=\frac{4\sin^2x}{2\cos^2x}$$

$$\Rightarrow\ \cos^4x = 2\sin^2x + \sin^4x \Rightarrow \sin x=\pm\frac{1}{2}$$

Hence, there will be 8 solutions in $[-2\pi, 2\pi]$

2. (b):

$$\lim_{x\to\infty}\frac{\left\{(1+x)^{2013}+(2+x)^{2013}+\dots+(2013+x)^{2013}\right\}}{x^{2013}+2013^{2013}}$$

$$=\lim_{x\to\infty}\frac{x^{2013}\left\{\left(\frac{1}{x}+1\right)^{2013}+\left(\frac{2}{x}+1\right)^{2013}+\dots\dots\dots+\left(\frac{2013}{x}+1\right)^{2013}\right\}}{x^{2013}\left(1+\left(\frac{2013}{x}\right)^{2013}\right)}$$

$$=\frac{(0+1)^{2013}+(0+1)^{2013}+\dots+(0+1)^{2013}}{1+0}=2013$$

[As $x\to\infty$, $1/x\to 0$]

3. (d): Let $x = z^3 \Rightarrow dx = 3z^2dz$

$$\int\frac{3z^2dz}{z^2-z}=3\int\frac{zdz}{z-1}=3\int\frac{(z-1)+1}{z-1}dz$$

$$=3\left[z+\log|z-1|\right]+c=3\left[\sqrt[3]{x}+\log|\sqrt[3]{x}-1|\right]+c$$

$\Rightarrow\ \alpha=\beta=\gamma=3\Rightarrow\alpha,\beta,\gamma$ are in A.P. and G.P. both.

4. (c) : $\because f(x)$ is an identity function $\therefore f(x) = x$

$\therefore$ Given equation becomes

$$\sum_{r=1}^{3}\{x-(2012+r)\}^{-1}=0$$

$$\Rightarrow\ \frac{1}{x-2013}+\frac{1}{x-2014}+\frac{1}{x-2015}=0$$

$\Rightarrow$ $(x-2014)(x-2015)+(x-2015)(x-2013)$ $+(x-2013)(x-2014)=0 \Rightarrow g(x)=0$ [say]

$\therefore$ $g(2013)=(-1)(-2)=2>0,\ g(2014)=(-1)(1)=-1<0$ and $g(2015)=(2)(1)=2>0$

As $g(x)$ changes sign between 2013 & 2014, 2014 & 2015, the equation $g(x)=0$ has roots between them.

$\therefore$ Both roots are real and different.

5. **(d):** Let $\dfrac{\sin x}{\cos A}=\dfrac{\sin y}{\cos B}=\dfrac{\sin z}{\cos C}=k$

$\therefore$ $\sin x = k\cos A$ ...(i)

$$\therefore \sum\left(\frac{\sin^2 A-\cos^2 x}{\sin x-\cos A}\right)=\sum\left(\frac{1-\cos^2 A-1+\sin^2 x}{\sin x-\cos A}\right)$$

$$=\sum\left(\frac{\sin^2 x-\cos^2 A}{\sin x-\cos A}\right)$$

$=\Sigma(\sin x+\cos A)=\Sigma\sin x+\Sigma\cos A$

$$=\frac{\left(\sum\sin x+\sum\cos A\right)\left(\sum\sin x-\sum\cos A\right)}{\sum\sin x-\sum\cos A}$$

$$=\frac{\left(\sum\sin x\right)^2-\left(\sum\cos A\right)^2}{\sum(\sin x-\cos A)}$$

6. **(d):** $\displaystyle\sum_{r=0}^{\infty}x^r(1+x+x^2)=\sum_{r=0}^{\infty}p_r x^r$

$\Rightarrow$ $(1+x+x^2)(1+x+x^2+x^3+\ldots \text{ to } \infty)$ $=p_0+p_1x+p_2x^2+p_3x^3+\ldots \text{ to } \infty$

$$\Rightarrow (1+x+x^2)\left(\frac{1}{1-x}\right)=p_0+p_1x+p_2x^2+\ldots+\text{ to }\infty$$

$$\Rightarrow p_0+p_1x+p_2x^2+p_3x^3+\ldots \text{ to } \infty = (1-x)^{-2}(1-x^3)$$

$=(1+2x+3x^2+4x^3+\ldots \text{ to } \infty)(1-x^3)$

$\therefore$ On comparing, we get

$p_0=1,\ p_1=2,\ p_2=3=p_3=p_4=p_5=\ldots\ldots..$

$$\therefore \sum_{r=0}^{671}p_r=p_0+p_1+p_2+p_3+\ldots+p_{671}$$

$=1+2+(3+3+3+\ldots \text{ to } 670 \text{ terms})$

$=3\times 671=2013$

7. **(b):** Let $I=\displaystyle\int\frac{dx}{(1+x\tan x)^2}$

$$=\int\frac{\cot^2 x\,dx}{(\cot x+x)^2}=\int\frac{-(1-\operatorname{cosec}^2 x)dx}{(\cot x+x)^2}$$

$$=-\int\frac{d(x+x\cot x)}{(x+\cot x)^2}=\frac{1}{x+\cot x}+c\Rightarrow f(x)=\cot x$$

8. **(d):** For non-trivial solution, we must have

$$\begin{vmatrix}\sin^2\alpha & 1 & 1\\ 1 & \sin^2\beta & 1\\ 1 & 1 & \sin^2\gamma\end{vmatrix}=0\Rightarrow\begin{vmatrix}p & 1 & 1\\ 1 & q & 1\\ 1 & 1 & r\end{vmatrix}=0$$

(where $p=\sin^2\alpha, q=\sin^2\beta, r=\sin^2\gamma$)

$$\Rightarrow\begin{vmatrix}p & 1 & 1\\ 1-p & q-1 & 0\\ 1-p & 0 & r-1\end{vmatrix}=0\left[\begin{matrix}R_2\to R_2-R_1,\\ R_3\to R_3-R_1\end{matrix}\right]$$

$\Rightarrow$ $p(q-1)(r-1)-(1-p)(r-1)+1\{0-(1-p)(q-1)\}=0$

$$\Rightarrow \frac{(p-1)+1}{p-1}+\frac{1}{q-1}+\frac{1}{r-1}=0$$

$$\Rightarrow \frac{1}{1-\sin^2\alpha}+\frac{1}{1-\sin^2\beta}+\frac{1}{1-\sin^2\gamma}=1$$

$$\Rightarrow \sec^2\alpha+\sec^2\beta+\sec^2\gamma=1\Rightarrow\sum(1+\tan^2\alpha)=1$$

$\Rightarrow$ $\sum\tan^2\alpha=-2<0$, which is impossible

9. **(c):** $\because$ $256\ a_1a_2a_3a_4\ge\left(\displaystyle\sum_{r=1}^{4}a_r\right)^4$

$\because$ $4\sqrt[4]{a_1a_2a_3a_4}\ge a_1+a_2+a_3+a_4$

$$\Rightarrow \frac{a_1+a_2+a_3+a_4}{4}\le\sqrt[4]{a_1a_2a_3a_4}\Rightarrow \text{A.M.}\le\text{G.M.}$$

But, A.M $\ge$ G.M $\Rightarrow$ A.M = G.M

$\Rightarrow$ $a_1=a_2=a_3=a_4$ ...(i)

Also, $502a_1+503a_2+504a_3+505\ a_4=2014$

$\therefore$ $2014\ a_1=2014\Rightarrow a_1=1=a_2=a_3=a_4$

Now, $\displaystyle\sum_{r=1}^{4}a_r^r=a_1+a_2^2+a_3^3+a_4^4=1+1+1+1=4$

10. **(d):** $\because$ $x^2+y^2=t-\dfrac{1}{t}$ and $x^4+y^4=t^2+\dfrac{1}{t^2}$

$$\Rightarrow x^4+y^4+2x^2y^2=t^2+\frac{1}{t^2}-2$$

and $x^4+y^4=t^2+\dfrac{1}{t^2}$

$\Rightarrow$ $x^2y^2=-1$, which is not possible for $x, y\in R$.

$\therefore$ (a), (b) and (c) are not possible.

11. **(a,b,c):** $\because$ $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

Let θ be the angle between $\vec{a}\ \&\ \vec{c}$ and $\vec{b}\ \&\ \vec{c}$.

$$\therefore \cos\theta=\frac{\vec{a}\cdot\vec{c}}{|\vec{a}||\vec{c}|}=\vec{a}\cdot\vec{c} \qquad \text{...(i)}$$

Similarly, $\cos\theta = \vec{b}\cdot\vec{c}$ ...(ii)

$\because\ \ \vec{a}\perp\vec{b}\ \ \therefore\ \ \vec{a}\cdot\vec{b}=0$

Also, $\vec{c} = p\vec{a} + q\vec{b} + r(\vec{a}\times\vec{b})$

$\therefore\ \ \vec{a}\cdot\vec{c} = p\vec{a}\cdot\vec{a} + q\,\vec{a}\cdot\vec{b} + r\,\vec{a}\cdot(\vec{a}\times\vec{b})$

$= p + 0 + 0 = p \Rightarrow \cos\theta = p\ \ \therefore\ \ |p| = |\cos\theta| \le 1$

Similarly, $\cos\theta = q \Rightarrow |q| \le 1$

Also, $p = q$.

12. (a, c) : $\because$ Roots are complex.

$\therefore\ \ D < 0 \Rightarrow (a+b+c)^2 - 4(ab+bc+ca) < 0$

$\Rightarrow\ \ (a+c)^2 + b^2 + 2b(a+c) - 4b(a+c) < 4ac$

$\Rightarrow\ \ (a+c-b)^2 < 4ac \Rightarrow -2\sqrt{ac} < a+c-b < 2\sqrt{ac}$

$\Rightarrow\ \ (\sqrt{a}+\sqrt{c})^2 > b \Rightarrow \sqrt{a}+\sqrt{c} > \sqrt{b} \Rightarrow$ (a) is correct.

$$\Rightarrow\ \ \sqrt{a}+\sqrt{c}-\sqrt{b} > 0 \Rightarrow \frac{1}{\sqrt{bc}}+\frac{1}{\sqrt{ab}}-\frac{1}{\sqrt{ca}} > 0$$

Similarly,

$$\frac{1}{\sqrt{bc}}+\frac{1}{\sqrt{ca}}-\frac{1}{\sqrt{ab}} > 0 \text{ and } \frac{1}{\sqrt{ca}}+\frac{1}{\sqrt{ab}}-\frac{1}{\sqrt{bc}} > 0$$

$\therefore$ On multiplying,

$$\left(\frac{1}{\sqrt{ab}}+\frac{1}{\sqrt{bc}}-\frac{1}{\sqrt{ca}}\right)\left(\frac{1}{\sqrt{bc}}+\frac{1}{\sqrt{ca}}-\frac{1}{\sqrt{ab}}\right)\left(\frac{1}{\sqrt{ca}}+\frac{1}{\sqrt{ab}}-\frac{1}{\sqrt{bc}}\right) > 0$$

$\Rightarrow$ (c) is correct.

13. (a, c, d) : $\because\ a\sin\theta + b\cos\theta = c$ ...(i)

& $a\,\mathrm{cosec}\,\theta + b\sec\theta = c$ ...(ii)

$\therefore$ On multiplying, (i) and (ii), we get

$$a^2+b^2+ab\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)=c^2$$

$$\Rightarrow\ a^2+b^2+2ab\left(\frac{1}{\sin 2\theta}\right)=c^2\ \therefore\ \sin 2\theta = \frac{2ab}{c^2-a^2-b^2}$$

Also, $a\sin\theta + b\cos\theta = a\,\mathrm{cosec}\,\theta + b\sec\theta$

$$\Rightarrow\ a\left(\frac{1}{\sin\theta}-\sin\theta\right)+b\left(\frac{1}{\cos\theta}-\cos\theta\right)=0$$

$$\Rightarrow\ a\frac{\cos^2\theta}{\sin\theta}+b\frac{\sin^2\theta}{\cos\theta}=0\ \therefore\ a\cos^3\theta + b\sin^3\theta = 0$$

From (i), $b\cos\theta = c - a\sin\theta$ ...(iii)

From (ii), $b\sec\theta = c - a\,\mathrm{cosec}\,\theta$ ...(iv)

From (iii) & (iv), $b^2 = c^2 + a^2 - ac(\sin\theta + \mathrm{cosec}\theta)$

$$\therefore\ \sin\theta + \mathrm{cosec}\,\theta = \frac{a^2+c^2-b^2}{ac}$$

14. (b,c,d) :

$\because$ If $x \to 0^+$, $2013^x \in (1, 2) \Rightarrow \{2013^x\} = 2013^x - 1$

And, $x \to 0^-$, $2013^x \in (0, 1) \Rightarrow \{2013^x\} = 2013^x$

$$\lim_{x\to0^+} f(2013^x) = \lim_{x\to0^+}(2013^x)^{\frac{1}{\{2013^x\}}}$$

$$= \lim_{x\to0^+}(2013^x)^{\frac{1}{2013^x-1}}$$

$$= \lim_{h\to0}(2013^h)^{\frac{1}{2013^h-1}}\quad (1^\infty \text{ form})$$

$$= e^{\lim\limits_{h\to0}\left[(2013^h-1)\cdot\frac{1}{(2013^h-1)}\right]} = e$$

And,

$$\lim_{x\to0^-} f(2013^x) = \lim_{x\to0^-}\left(2013^x\right)^{\frac{1}{\{2013^x\}}}$$

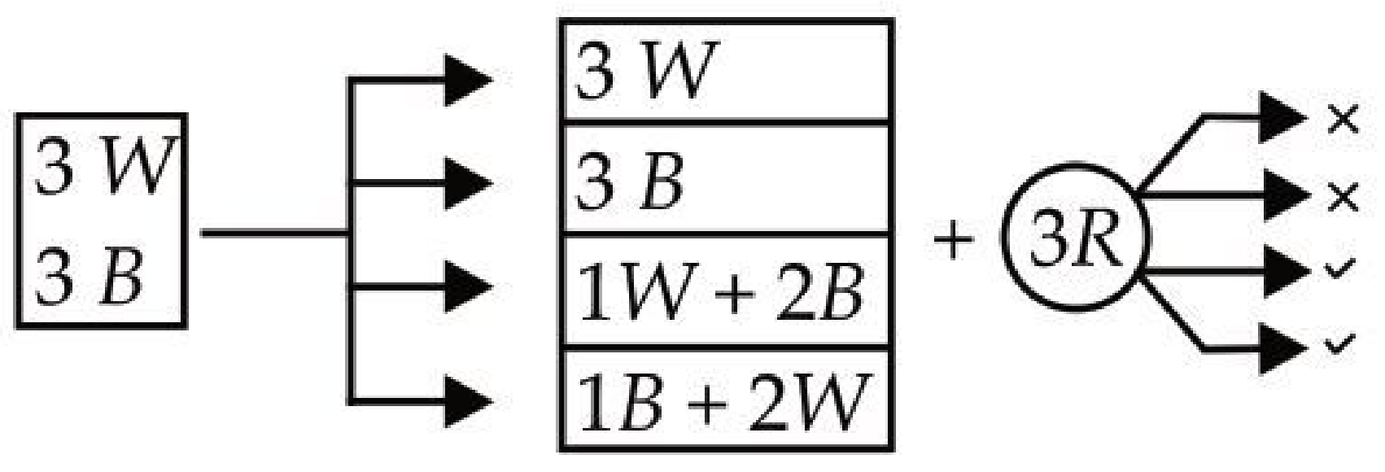
$$= \lim_{x\to0^-}\left(2013^x\right)^{\frac{1}{2013^x}} = \lim_{h\to0}\left(2013^{0-h}\right)^{\frac{1}{\{2013^{0-h}\}}}$$

$$= \lim_{h\to0}\left(2013^{-h}\right)^{\frac{1}{2013^{-h}}} = 1^1 = 1$$

$\because$ R.H.L $\ne$ L.H.L

$\Rightarrow\ \lim\limits_{x\to0} f(2013^x)$ does not exist

15. (a, b) :



Clearly, required probability

$$= 2\left[\left(\frac{{}^3C_1\times{}^3C_2}{{}^6C_3}\right)\times\left(\frac{{}^1C_1\times{}^2C_1\times{}^3C_1}{{}^6C_3}\right)\right]$$

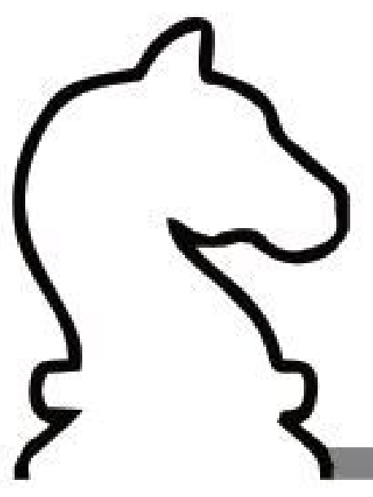
$$= \frac{2\cdot3\cdot3\cdot1\cdot2\cdot3}{20\cdot20} = \frac{27}{100} = 27\%$$

◆◆

Monthly Test Drive CLASS XI ANSWER KEY

1.	(b)	**2.**	(d)	**3.**	(b)	**4.**	(a)	**5.**	(c)
6.	(d)	**7.**	(c)	**8.**	(b,c)	**9 .**	(b,c)	**10.**	(a,b)
11.	(b,d)	**12.**	(d)	**13.**	(a,b,d)	**14.**	(a)	**15.**	(c)
16.	(b)	**17.**	(0)	**18.**	(2)	**19.**	(1)	**20.**	(6)

Challenging PROBLEMS ON JEE



Single Option Correct Type

1. The number of real solutions x of the equation $x^{10} - x^8 + 8x^6 - 24x^4 + 32x^2 - 48 = 0$ is
(a) 0 (b) 1 (c) 2 (d) 4

2. For complex number z with $|z| = 1$, the maximum value of $|z^3 - z + 2|$ is
(a) 13 (b) 16 (c) 20 (d) 24

3. Let $a, b, c \in R^+$ and $a + b + c = abc$, then minimum value of $\sqrt{1+\frac{1}{a^2}}+\sqrt{1+\frac{1}{b^2}}+\sqrt{1+\frac{1}{c^2}}$ is
(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $2\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

4. The number of values of m for which the system of equations $x^2 + y^2 = 1$, $x^2 = 2^{|x|} + |x| - y - m$ has a unique solution is
(a) 0 (b) 1 (c) 2 (d) 4

5. $\sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)(2k+1)3^k} = 3\log\left(\frac{3}{4}\right)+\lambda$ where λ is
(a) $\frac{\pi}{\sqrt{3}}$ (b) $\frac{\pi}{3}$ (c) $\pi\sqrt{3}$ (d) 3π

6. The locus of point P in the plane of square $ABCD$ (side length 1 cm) such that maximum $\{PA, PC\}$ $= \frac{1}{\sqrt{2}}(PB + PD)$ encloses an area of
(a) $\frac{1}{2}$ (b) $\frac{\pi}{2}$ (c) 2 (d) 2π

7. $\int\frac{(2\sin x+1)dx}{(2+\sin x)^2} =$
(a) $\frac{\cos x}{2+\sin x}+c$ (b) $\frac{-\cos x}{2+\sin x}+c$
(c) $\frac{\sin x}{2+\sin x}+c$ (d) $\frac{-\sin x}{2+\sin x}+c$

8. O is the centre, AB and BC are two diagonals of the adjacent faces of a rectangular box. If angles AOB, BOC and COA and α, β, θ then $\cos\alpha + \cos\beta + \cos\theta =$
(a) 1.5 (b) 1 (c) 0 (d) –1

9. The number of non-empty subsets of $\{1, 2, 3, ..., 12\}$ having the property that the sum of the largest element and the smallest element is 13, is
(a) 2540 (b) 1365 (c) 1250 (d) 1100

10. We call a number, T-number if its middle digit is larger than any other digit. For example 254 is a T-number. How many such 3-digit T-numbers are there?
(a) 120 (b) 125 (c) 240 (d) 250

Comprehension Type

Paragraph for Q. No. 11 and 12

Let M be the centroid of ΔABC with $\angle AMB = 2\angle ACB$

11. $AC^4 + BC^4 - AB^4 =$
(a) $AC^2 \cdot BC^2$ (b) $AC^2 \cdot AB^2$
(c) $AB^2 \cdot BC^2$ (d) $BC^2 \cdot AM^2$

12. $\angle ACB$
(a) ≥ 60° (b) ≤ 60° (c) ≥ 90° (d) < 90°

Integer Answer Type

13. If $|x + y| + |x - y| = 2$ then maximum possible value of $x^2 - 6x + y^2$ is

14. Let $x, y \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ and $x^3 + \sin x = 2a$ and $4y^3 + \frac{1}{2}\sin 2y + a = 0$ then $\cos(x + 2y) =$ _____.

15. If $\frac{\tan 1}{\cos 2}+\frac{\tan 2}{\cos 4}+....+\frac{\tan 1024}{\cos 2048} = \tan\lambda - \tan\mu$, then $\left[\frac{\lambda-\mu}{1000}\right]$ ([·] denotes greatest integer function) = _____.

16. For complex number z, $\arg(z^2 - 4) = \frac{5\pi}{6}$ and $\arg(z^2 + 4) = \frac{\pi}{3}$, then $|z| =$_____.

17. In ΔABC, H is its orthocentre then $\frac{AH^2 + BH^2 + BC^2 + AC^2}{CH^2 + AB^2} =$ _____.

18. Let $y = x^4 + 9x^3 + ax^2 + 9x + 4$ contains four collinear points, then maximum value of $\left[\frac{a}{5}\right]$ is([·] denotes greatest integer function)______.

SOLUTIONS

1. (c) : Putting $x^2 = y$, the given equation becomes

$$y^5 - y^4 + 8y^3 - 24y^2 + 32y - 48 = 0$$

i.e., $(y - 2)(y^4 + y^3 + 10y^2 - 4y + 24) = 0$

$\Rightarrow$ $(y - 2)(y^4 + y^3 + 9y^2 + (y - 2)^2 + 20) = 0$

Hence, $y = 2$ is only required solution $\Rightarrow x = \pm\sqrt{2}$

2. (a) : Putting $z = x + iy \Rightarrow x^2 + y^2 = 1$, $x, y \in [-1, 1]$

$f(z) = |z^3 - z + 2|$ becomes $g(x) = 16x^3 - 4x^2 - 16x + 8$

For maximum, put $g'(x) = 0 \Rightarrow x = \frac{-1}{2}, \frac{2}{3}$

$\therefore\ g\left(\frac{-1}{2}\right) = 13$ is maximum

3. (c) : Let $a = \tan x$ etc. Then, according to question

$\Sigma\tan x = \Pi\tan x \Rightarrow x + y + z = \pi$

So, given expression $= \operatorname{cosec}x + \operatorname{cosec}y + \operatorname{cosec}z$

$$\geq 3\operatorname{cosec}\left(\frac{x+y+z}{3}\right)$$

[According to Jensen's inequality. cosec function is convex in $(0, \pi)$]

$$= 2\sqrt{3}$$

4. (b) : Notice that if (x, y) is a solution, then $(-x, y)$ is also a solution, hence, for unique solution $x = 0$.

Hence $y = 1, -1 \Rightarrow m = 0, 2$

But for $m = 2$, there are two solutions $(x, y) = (1, 0)$, $(0, -1)$.

Hence, only for $m = 0$, there exists a unique solution.

5. (a) : Given sum, $S = \sum_{k=0}^{\infty} \frac{(-1)^k}{3^k}\left[\frac{2}{2k+1} - \frac{1}{k+1}\right]$

$$= 2\sum_{k=0}^{\infty}\frac{(-1)^k}{3^k}\left(\int_0^1 x^{2k}\,dx\right) - \sum_{k=0}^{\infty}\frac{(-1)^k}{3^k}\left(\int_0^1 x^k\,dx\right)$$

$$= 2\cdot\int_0^1\sum_{k=0}^{\infty}\left(\frac{-x^2}{3}\right)^k dx - \int_0^1\sum_{k=0}^{\infty}\left(\frac{-x}{3}\right)^k dx$$

$$= 2\cdot\int_0^1 \frac{1}{1+\frac{x^2}{3}}\,dx - \int_0^1\frac{1}{1+\frac{x}{3}}\,dx \quad \text{(sum of infinite G.P.)}$$

$$\therefore\ S = \frac{\pi}{\sqrt{3}} + 3\log(3/4)$$

6. (b) : Let max. $\{PA, PC\} = PA$, then according to question, $\sqrt{2}PA = PB + PD$

i.e. $BD \cdot PA = AD \cdot PB + AB \cdot PD$

Hence, by converse of ptolemy's theorem, it follows that $PDAB$ is cyclic quadrilateral. *i.e.* locus of P is circumcircle of square $ABCD$ with radius $= \frac{1}{\sqrt{2}}$.

7. (b) : Given integral $= \int\frac{(2\tan x\sec x + \sec^2 x)}{(\tan x + 2\sec x)^2}dx$

Now, take $\tan x + 2\sec x = t$

8. (d) : Let $O(0, 0, 0)$ and edges be along co-ordinate axes and let $A(a, -b, -c)$, $B(a, b, c)$, $C(-a, b, -c)$ then $\cos\alpha + \cos\beta + \cos\theta$

$$= \frac{(a^2 - b^2 - c^2) + (-a^2 + b^2 - c^2) + (-a^2 - b^2 + c^2)}{a^2 + b^2 + c^2} = -1$$

9. (b) : Let t be the smallest element, then $(13 - t)$ is the largest element *i.e.*, $13 - t \geq t \Rightarrow t \leq 6$.

Now, the set can contain any combination of elements from $(t + 1)$ to $(12 - t)$. There are $(12 - 2t)$ such elements.

SAMURAI SUDOKU

ANSWER · JUNE 2021

3	6	9	4	8	5	2	7	1				2	9	4	6	1	7	5	3	8
7	2	4	9	6	1	5	8	3				5	1	3	8	4	9	7	6	2
5	1	8	3	2	7	6	4	9				7	6	8	2	5	3	4	1	9
1	3	2	8	9	6	7	5	4				9	2	1	4	7	8	3	5	6
4	8	5	7	3	2	1	9	6				4	8	6	5	3	1	9	2	7
6	9	7	1	5	4	3	2	8				3	7	5	9	6	2	1	8	4
9	5	6	2	4	3	8	1	7	2	4	3	6	5	9	3	2	4	8	7	1
8	7	3	5	1	9	4	6	2	1	5	9	8	3	7	1	9	6	2	4	5
2	4	1	6	7	8	9	3	5	7	8	6	1	4	2	7	8	5	6	9	3
						5	4	1	9	2	7	3	8	6						
						2	9	6	8	3	1	4	7	5						
						7	8	3	5	6	4	9	2	1						
6	5	1	4	7	8	3	2	9	6	7	8	5	1	4	7	2	3	6	8	9
2	8	7	9	1	3	6	5	4	3	1	2	7	9	8	6	4	5	1	3	2
3	9	4	2	6	5	1	7	8	4	9	5	2	6	3	1	8	9	5	7	4
4	1	9	3	8	2	7	6	5				6	3	7	2	5	8	9	4	1
5	7	2	6	4	1	8	9	3				4	5	2	3	9	1	8	6	7
8	3	6	5	9	7	4	1	2				1	8	9	4	7	6	3	2	5
9	4	3	7	2	6	5	8	1				3	7	1	5	6	2	4	9	8
7	2	8	1	5	4	9	3	6				9	4	6	8	1	7	2	5	3
1	6	5	8	3	9	2	4	7				8	2	5	9	3	4	7	1	6

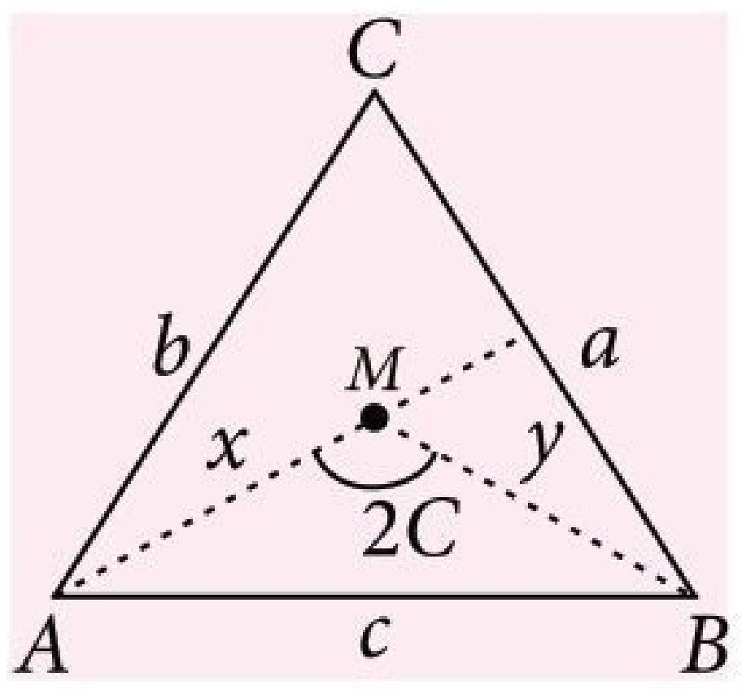
So, there are $2^{(12-t)-(t+1)+1} = 2^{12-2t}$ such combinations.
Hence, total required number of solution sets

$$=\sum_{t=1}^{6} 2^{12-2t} = 1365$$

10. (c) : Following 3 types of T-numbers are possible.
(i) xyz form $(x \neq 0) \rightarrow {}^9C_2$
(ii) xyz form $(x \neq z,\ z \neq 0) \rightarrow 2 \times {}^9C_3$
(iii) xyz form $(x \neq 0,\ y \neq 0) \rightarrow {}^9C_2$

$\therefore$ Required 3-digit T-numbers $= {}^9C_2 + 2 \cdot {}^9C_3 + {}^9C_2 = 240$

(11-12) :



In ΔAMB,

$$\cos 2C = \frac{x^2+y^2-c^2}{2xy} \quad \text{...(i)}$$

and area of $\Delta AMB = \frac{1}{3}$ area of ΔABC

$$\Rightarrow \frac{1}{2}xy\cdot\sin 2C = \frac{1}{3}\Delta \quad \Rightarrow \quad xy = \frac{2\Delta}{3\sin 2C} \quad \text{...(ii)}$$

Now, $x = \frac{2}{3}$(length of median from A)

$$= \frac{2}{3}\cdot\frac{1}{2}\sqrt{2b^2+2c^2-a^2} \quad \text{...(iii)}$$

Similarly, $y = \frac{2}{3}\cdot\frac{1}{2}\sqrt{2c^2+2a^2-b^2}$ ...(iv)

Simplifying (i), (ii), (iii), (iv), we have

$$c^4 = a^4 + b^4 - a^2b^2$$

And, it is clear that, $a^4 + b^4 - a^2b^2 \geq (a^2 + b^2 - ab)^2$
i.e. $a^2 + b^2 - 2ab\cos C = c^2 \geq a^2 + b^2 - ab$

i.e., $\cos C \leq \frac{1}{2} \quad \Rightarrow \quad \angle C \geq 60°$

11. (a) **12. (a)**

13. (8) : $|x+y| + |x-y| = 2 \Rightarrow$ It is square with vertices at $(\pm 1, \pm 1)$
$x^2 - 6x + y^2 = (x-3)^2 + y^2 - 9$, which is the distance of any point on the square from $(3, 0)$.

14. (1) : Replacing $2y$ with z and taking $f(x) = x^3 + \sin x$, the given equation becomes
$f(x) = f(-z) \ \Rightarrow \ x = -z$ as $f(x)$ is odd and increasing in $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

So, $x = -2y$
$\therefore \cos(x + 2y) = \cos(0) = 1$

15. (2) : Using, $\frac{\tan\theta}{\cos 2\theta} = \tan 2\theta - \tan\theta$, the given sum becomes
$(\tan 2 - \tan 1) + (\tan 4 - \tan 2) + ... + (\tan 2048 - \tan 1024)$
$= \tan 2048 - \tan 1$

16. (2) : Let $O(0, 0)$, $B(z^2)$, $A(z^2 - 4)$, $C(z^2 + 4)$, then $AB = BC = 4$ and $\angle OAX = \frac{5\pi}{6}$, $\angle OCX = \frac{\pi}{3}$, then ΔOBC is equilateral.

$$\text{So, } z^2 = 4\left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right]$$

i.e., $z = \pm(1 + i\sqrt{3}) \Rightarrow |z| = 2$

17. (2) : If R is the circumradius of the triangle, then
$AH^2 + BC^2 = 4R^2 = BH^2 + AC^2 = CH^2 + AB^2$

18. (6) : According to question, $y = 0$ has four distinct roots. So, $\frac{d^2y}{dx^2} = 12x^2 + 54x + 2a$ should have 2 real and distinct roots.

$$\Rightarrow D > 0 \Rightarrow a < \frac{243}{8}$$

$$\text{So, } \left[\frac{a}{5}\right]_{\max} = 6$$

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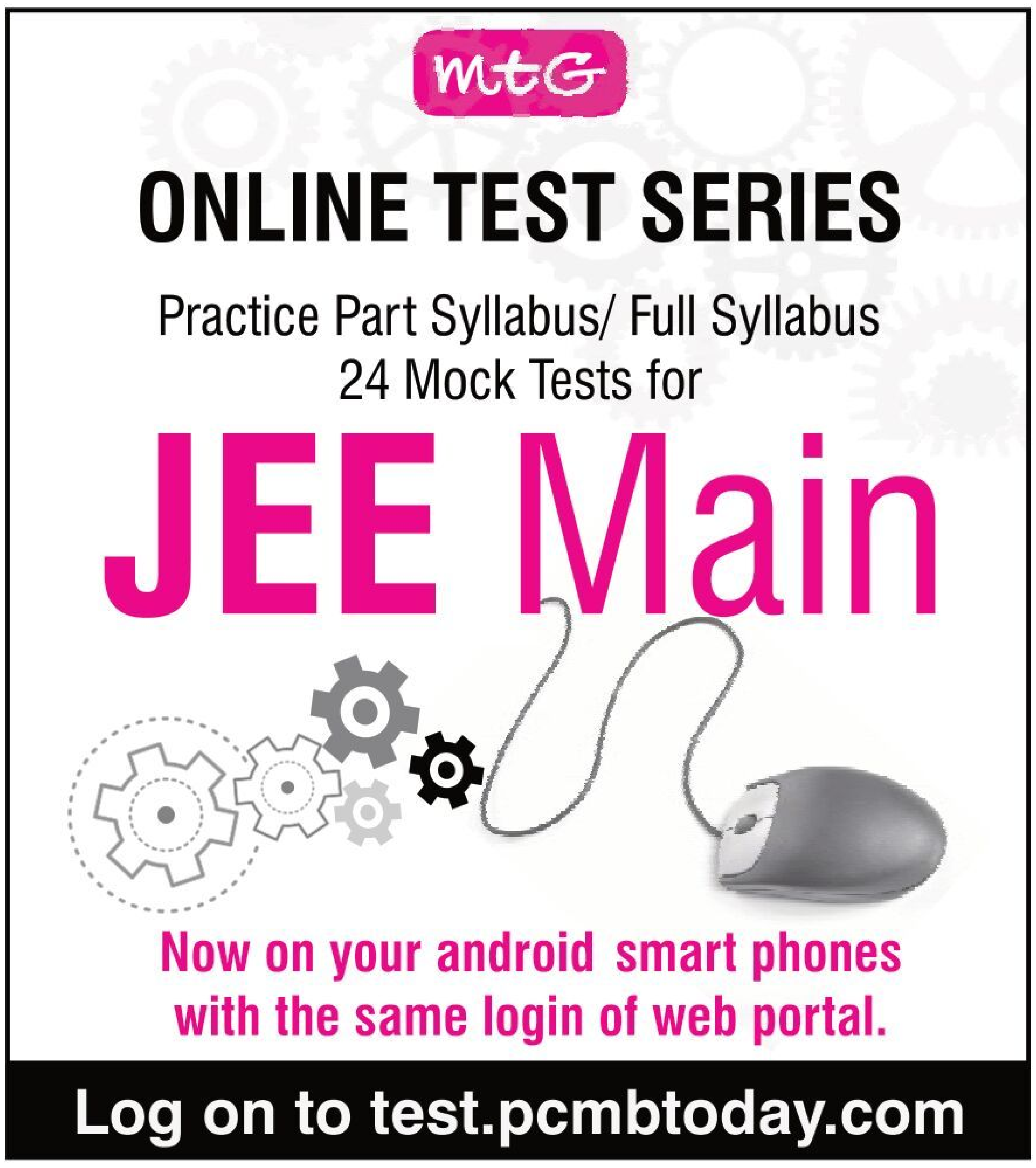
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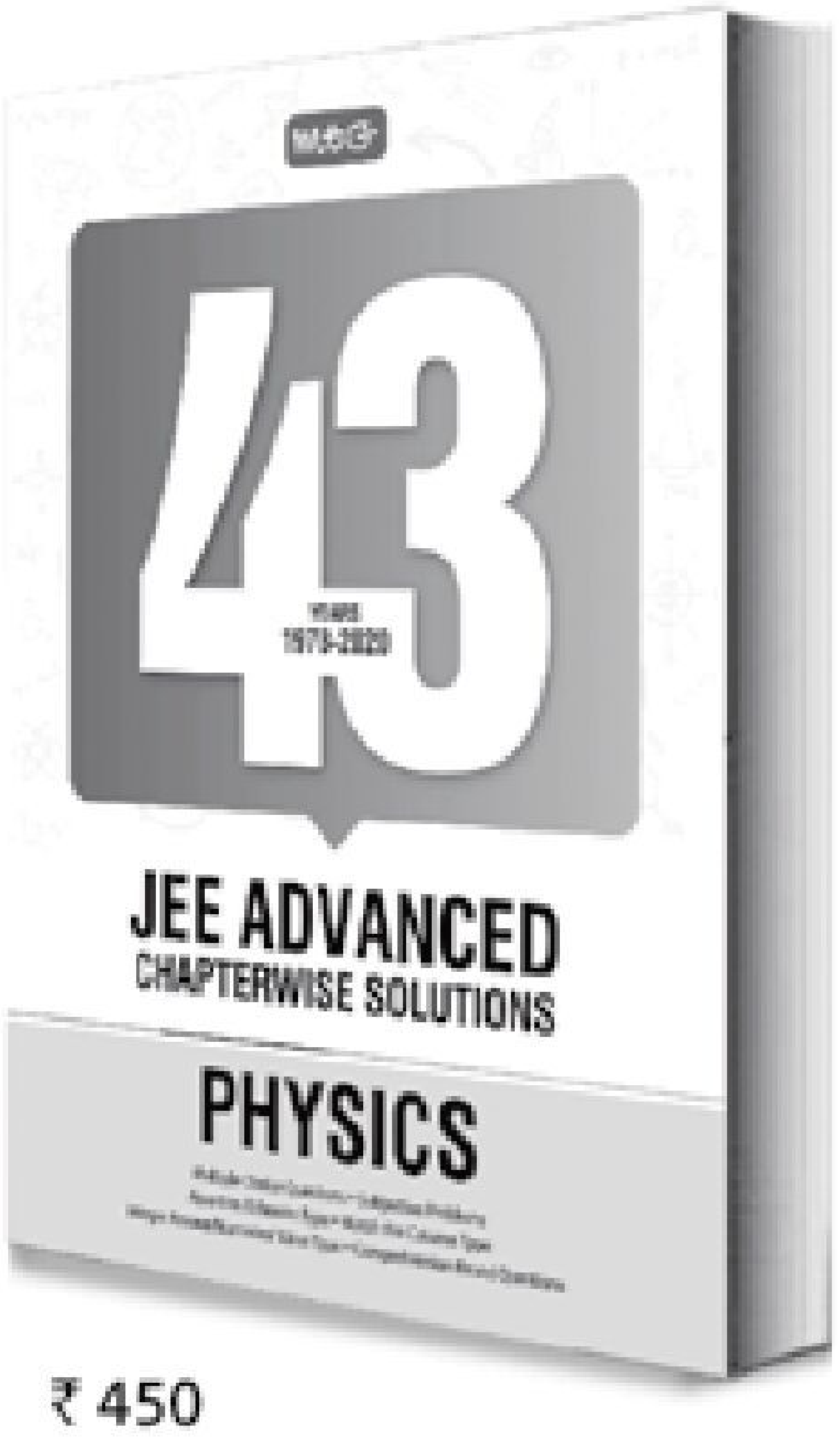
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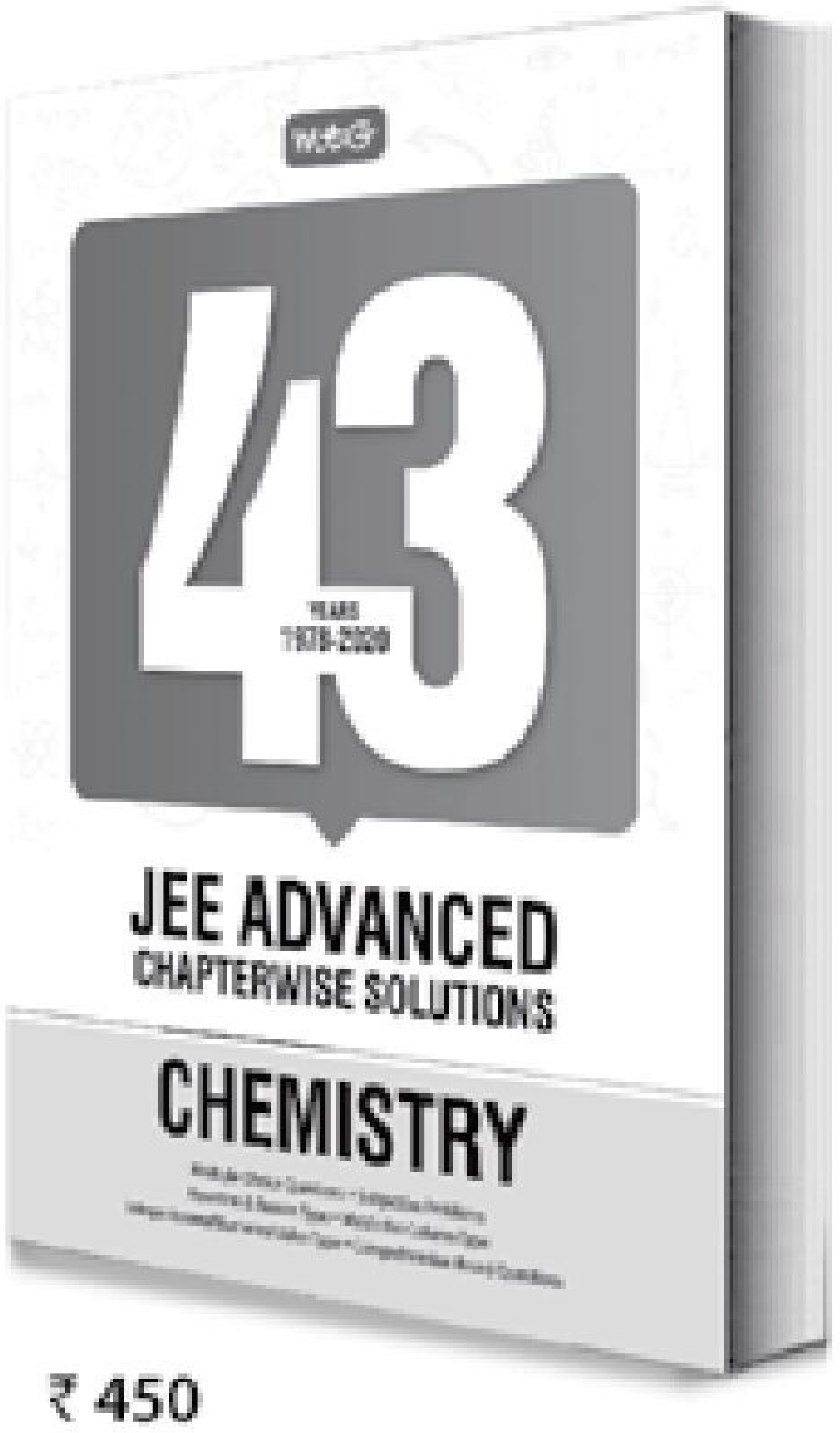
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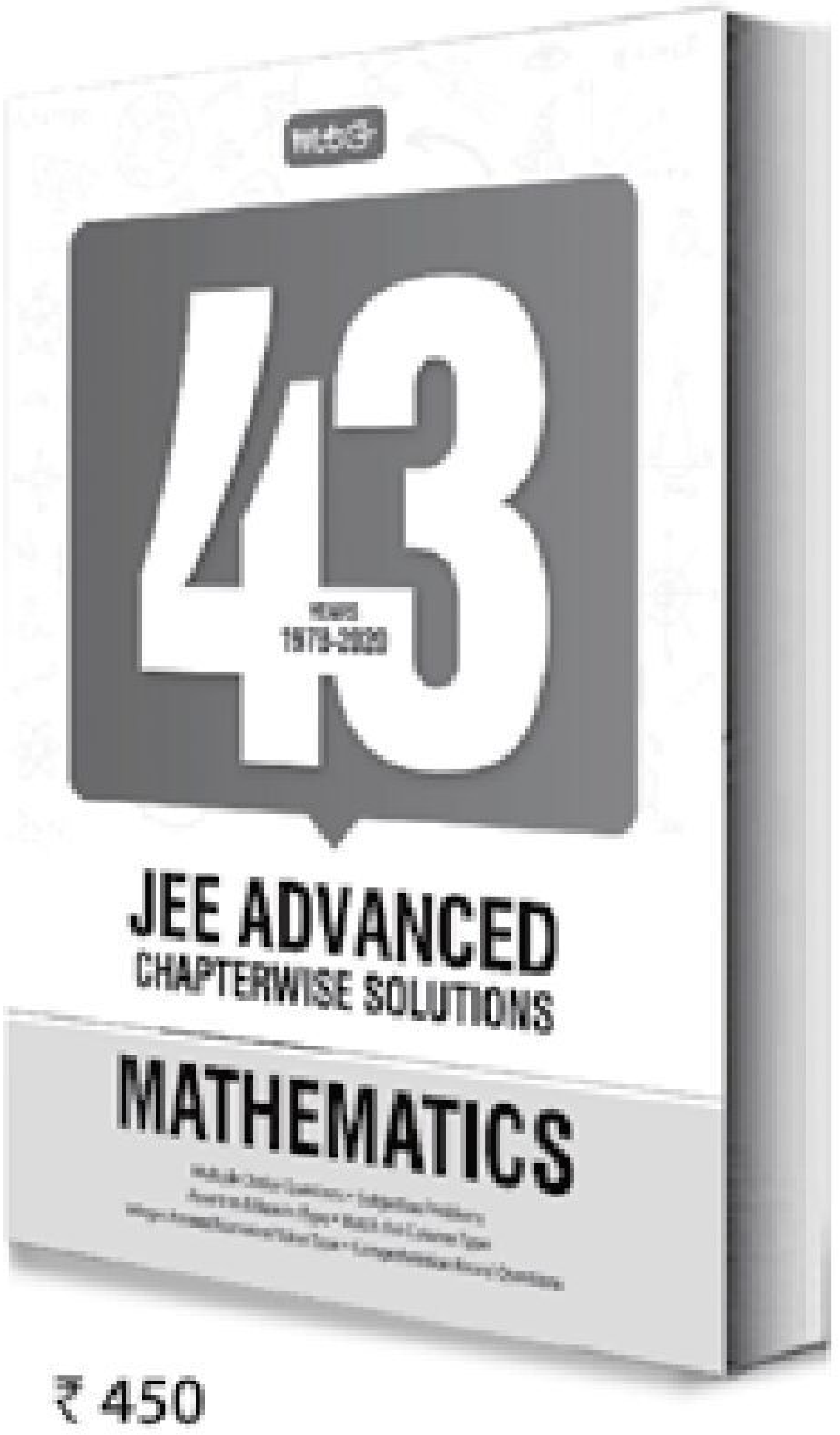
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OLYMPIAD CORNER

1. Let $a_1, a_2, ..., a_{100}$ be an ordered set of numbers. At each move it is allowed to choose any two numbers a_n, a_m and change them to the numbers

$$\frac{a_n^2}{a_m} - \frac{n}{m}\left(\frac{a_m^2}{a_n} - a_m\right) \text{ and } \frac{a_m^2}{a_n} - \frac{m}{n}\left(\frac{a_n^2}{a_m} - a_n\right)$$

respectively. Determine if it is possible, starting with the set with $a_i = \frac{1}{5}$ for i = 20, 40, 60, 80, 100 and a_i = 1 otherwise, to obtain a set consisting of integers only.

2. Does there exist a bijection f of

(a) a plane with itself

(b) three-dimensional space with itself such that for any distinct points A, B, line AB and line $f(A)\,f(B)$ are perpendicular?

3. Find all triplets (x, y, z) of natural numbers such that y is a prime number, y and 3 do not divide z, and $x^3 - y^3 = z^2$.

4. Prove that the equation $x^3 + y^3 + z^3 + t^3 = 1999$ has infinitely many integral solutions.

5. Determine the maximum value of λ such that if $f(x) = x^3 + ax^2 + bx + c$ is a cubic polynomial with all its roots non negative, then $f(x) \geq \lambda\,(x - a)^3$ for all $x \geq 0$. Find the equality condition.

6. Determine all functions $f: R - \{0, 1\} \to R$ satisfying the functional relation $f(x) + f\left(\frac{1}{1-x}\right) = \frac{2(1-2x)}{x(1-x)}$.

7. Show that $19^{93} - 13^{99}$ is a positive integer divisible by 162.

8. A sergeant appoints arbitrarily chosen 9 or 10 soldiers every night, out of a 33 member guard team, for duty. Determine the least number of days within which all guards can be on duty the same number of times. How can the guard duty be arranged to achieve this?

9. Find all primes p for which the quotient $\frac{(2^{p-1} - 1)}{p}$ is a square.

10. Prove that

$$1 < \frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + ... + \frac{1}{3001} < \frac{4}{3}$$

11. Three congruent circles have a common point O and lie inside a triangle such that each circle touches a pair of sides of the triangle. Prove that the incentre and the circumcentre of the triangle and the point O are collinear.

12. Find the number of permutations, $(P_1, P_2, ..., P_6)$, of $(1, 2, ..., 6)$ such that for any k, $1 \leq k \leq 5$, $(P_1, P_2, ..., P_k)$ does not form a permutation of $1, 2, ..., k$.
[That is, $P_1 \neq 1$; (P_1, P_2) is not a permutation of 1, 2, 3, etc.]

13. Let f be a bijective (one-one and onto) function from the set $A = \{1, 2, 3, ..., n\}$ to itself. Show that there is a positive integer $M > 1$ such that $f^M(i) = f(i)$ for each $i \in A$.
[f^M denotes the composite function $fofo$ .... repeated M times.]

SOLUTIONS

1. Suppose we change

$$a_n \text{ to } a_n' = \frac{a_n^2}{a_m} - \frac{n}{m}\left(\frac{a_m^2}{a_n} - a_m\right) \text{ and}$$

$$a_m \text{ to } a_m' = \frac{a_m^2}{a_n} - \frac{m}{n}\left(\frac{a_n^2}{a_m} - a_n\right)$$

Observe that $\frac{a_n'}{n} + \frac{a_m'}{m}$ equals

$$\left[\left(\frac{1}{n}\cdot\frac{a_n^2}{a_m} - \frac{1}{m}\cdot\frac{a_m^2}{a_n}\right) + \frac{a_m}{m}\right] + \left[\left(\frac{1}{m}\cdot\frac{a_m^2}{a_n} - \frac{1}{n}\cdot\frac{a_n^2}{a_m}\right) + \frac{a_n}{n}\right]$$

which equals $\frac{a_n}{n}+\frac{a_m}{m}$. Thus, the quantity $\sum_{i=1}^{100}\frac{a_i}{i}$ is invariant under the given operation. Originally, this sum equals $I_1=\sum_{i=1}^{99}\frac{a_i}{i}+\frac{1}{500}$.

When each of the numbers $\frac{a_1}{1},\frac{a_2}{2},\ldots,\frac{a_{99}}{99}$ is written as a fraction in lowest terms, none of their denominators are divisible by 125. On the other hand, 125 does divide the denominator of $\frac{1}{500}$. Thus when written as a fraction in lowest terms, I_1 must have a denominator divisible by 125.

Now suppose, by way of contradiction, that we could make all the numbers equal to integers $b_1, b_2, \ldots, b_{100}$ in that order. Then in $I_2=\sum_{i=1}^{100}\frac{b_i}{i}$, the denominator of each of the fraction $\frac{b_i}{i}$ is not divisible by 125. Thus, when I_2 is written as a fraction in lowest terms, its denominator is not divisible by 125 either. Then I_2 cannot possibly equal I_1, a contradiction. Therefore, we can never obtain a set consisting of integers only.

2. (a) Yes: simply rotate the plane 90° about some axis perpendicular to it. For example, in the xy-plane, we could map each point (x, y) to the point $(y, -x)$.

(b) Suppose such a bijection existed. Label the three-dimensional space with x-, y-, and z-axes. Given any point $P = (x_0, y_0, z_0)$, we also view it as the vector p from $(0, 0, 0)$ to (x_0, y_0, z_0). The given condition says that $(a - b)\cdot(f(a) - f(b)) = 0$ for any vectors a, b.

Assume without loss of generality that f maps the origin to itself otherwise, $g(p) = f(p) - f(0)$ is still a bijection and still satisfies the above equation. Plugging $b = (0, 0, 0)$ into the equation above, we have $a\cdot f(a) = 0$ for all a. Then the above equation reduces to $a\cdot f(b) + b\cdot f(a) = 0$.

Given any vectors a,b,c and any reals m, n, we then have

$$m(a\cdot f(b) + b\cdot f(a)) = 0$$

$$n(a\cdot f(c) + c\cdot f(a)) = 0$$

$$a\cdot f(mb + nc) + (mb + nc)\cdot f(a) = 0.$$

Adding the first two equations and subtracting the third gives $a\cdot(m\,f(b) + n\,f(c) - f(mb + nc)) = 0$.

Because this must be true for any vector a, we must have $f(mb + nc) = m\,f(b) + n\,f(c)$.

Therefore f is linear, and it is determined by how it transforms the unit vectors $i = (1, 0, 0)$, $j = (0, 1, 0)$, and $k = (0, 0, 1)$. If $f(i) = (a_1, a_2, a_3)$, $f(j) = (b_1, b_2, b_3)$ and $f(k) = (c_1, c_2, c_3)$, then for a vector x, we have

$$f(x)=\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}x$$

Applying $f(a)\cdot a = 0$ with $a = i, j, k$, we have
$a_1 = b_2 = c_3 = 0$
Then applying $a\cdot f(b) + b\cdot f(a)$ with $(a, b) = (i, j), (j, k), (k, i)$ we have $b_1 = -a_2$, $c_2 = -b_3$, $c_1 = -a_3$.
Setting $k_1 = c_2$, $k_2 = -c_1$, and $k_3 = b_1$, we find that
$f(k_1 i + k_2 j + k_3 k) = k_1 f(i) + k_2 f(j) + k_3 f(k) = 0$
Because f is injective and $f(0) = 0$, this implies that $k_1 = k_2 = k_3 = 0$.
Then $f(x) = 0$ for all x, contradicting the assumption that f was surjective. Therefore our original assumption was false, and no such bijection exists.

3. We rewrite the equation in the form
$(x - y)(x^2 + xy + y^2) = z^2$
Any common divisor of $x - y$ and $x^2 + xy + y^2$ also divides both z^2 and $(x^2 + xy + y^2) - (x + 2y)(x - y) = 3y^2$. Because z^2 and $3y^2$ are relatively prime by assumption, $x - y$ and $x^2 + xy + y^2$ must be relatively prime as well. Therefore, both $x - y$ and $x^2 + xy + y^2$ are perfect squares.

Writing $a=\sqrt{x-y}$, we have

$$x^2 + xy + y^2 = (a^2 + y)^2 + (a^2 + y)y + y^2$$
$$= a^4 + 3a^2y + 3y^2$$

and $4(x^2 + xy + y^2) = (2a^2 + 3y)^2 + 3y^2$

Writing $m=2\sqrt{x^2+xy+y^2}$ and $n=2a^2+3y$, we have

$$m^2 = n^2 + 3y^2 \quad\text{or}\quad (m - n)(m + n) = 3y^2,$$

so $(m - n, m + n) = (1, 3y^2), (y, 3y)$ or $(3, y^2)$.
In the first case, $2n = 3y^2 - 1$ and $4a^2 = 2n - 6y = 3y^2 - 6y - 1$
Hence, $a^2 \equiv 2 \pmod 3$, which is impossible.
In the second case, $n = y < 2a^2 + 3y = n$, a contradiction.
In the third case, we have

$$4a^2 = 2n - 6y = y^2 - 6y - 3 < (y - 3)^2.$$

When $y \ge 10$ we have $y^2 - 6y - 3 > (y - 4)^2$.
Hence, we must actually have $y = 2, 3, 5$, or 7. In this case, we have $a=\frac{\sqrt{y^2-6y-3}}{2}$, which is real only when $y = 7$, $a = 1$, $x = y + a^2 = 8$ and $z = 13$. This yields the unique solution $(x, y, z) = (8, 7, 13)$.

4. Observe that $(m - n)^3 + (m + n)^3 = 2m^3 + 6mn^2$. Now suppose we want a general solution of the form

$$(x, y, z, t)=\left(a-b, a+b, \frac{c}{2}-\frac{d}{2}, \frac{c}{2}+\frac{d}{2}\right)$$

for integers a, b and odd integers c, d. One simple solution to the given equation is $(x, y, z, t) = (10, 10, -1, 0)$, so we try setting $a = 10$ and $c = -1$. Then

$$(x, y, z, t)=\left(10-b, 10+b, -\frac{1}{2}-\frac{d}{2}, -\frac{1}{2}+\frac{d}{2}\right)$$

is a solution exactly when

$$(2000+60b^2)-\frac{1+3d^2}{4}=1999 \Leftrightarrow d^2-80b^2=1$$

The second equation is a Pell's equation with solution $(d_1, b_1) = (9, 1)$

We can generate infinitely many more solutions by setting $(d_{n+1}, b_{n+1}) = (9d_n + 80b_n, 9b_n + d_n)$ for $n = 1, 2, 3, \ldots$

This can be proven by induction, and it follows from a general recursion

$$(p_{n+1}, q_{n+1}) = (p_1p_n + q_1q_nD, p_1q_n + q_1p_n)$$

for generating solutions to $p^2 - Dq^2 = 1$ given a non trivial solution (p_1, q_1).

A quick check also shows that each d_n is odd. Thus because there are infinitely many solutions (b_n, d_n) to the Pell's equation (and with each d_n odd), there are infinitely many integral solutions

$$(x_n, y_n, z_n, t_n)=\left(10-b_n, 10+b_n, -\frac{1}{2}-\frac{d_n}{2}, -\frac{1}{2}+\frac{d_n}{2}\right)$$

to the original equation.

5. Let α, β, γ be the three roots. Without loss of generality, suppose that $0 \le \alpha \le \beta \le \gamma$. We have $x - a = x + \alpha + \beta + \gamma \ge 0$ and $f(x) = (x - \alpha)(x - \beta)(x - \gamma)$.

If $0 \le x \le \alpha$, then (applying the AM-GM inequality to obtain the first inequality below)

$$-f(x)=(\alpha-x)(\beta-x)(\gamma-x) \le \frac{1}{27}(\alpha+\beta+\gamma-3x)^3$$

$$\le \frac{1}{27}(x+\alpha+\beta+\gamma)^3=\frac{1}{27}(x-a)^3$$

so that $f(x) \ge -\frac{1}{27}(x-a)^3$.

Equality holds exactly when $\alpha - x = \beta - x = \gamma - x$ in the first inequality and $\alpha + \beta + \gamma - 3x = x + \alpha + \beta + \gamma$ in the second; that is, when $x = 0$ and $\alpha = \beta = \gamma$.

If $\beta \le x \le \gamma$, then (again applying the AM-GM inequality to obtain the first inequality below)

$$-f(x)=(x-\alpha)(x-\beta)(\gamma-x) \le \frac{1}{27}(x+\gamma-\alpha-\beta)^3$$

$$\le \frac{1}{27}(x+\alpha+\beta+\gamma)^3=\frac{1}{27}(x-a)^3$$

so that again $f(x) \ge -\frac{1}{27}(x-a)^3$.

Equality holds exactly when $x - \alpha = x - \beta = \gamma - x$ in the first inequality and $x + \gamma - \alpha - \beta = x + \alpha + \beta + \gamma$; that is, when $\alpha = \beta = 0$ and $\gamma = 2x$.

Finally, when $\alpha < x < \beta$ or $x > \gamma$ then

$$f(x) > 0 \ge -\frac{1}{27}(x-a)^3$$

Thus, $\lambda=-\frac{1}{27}$ works. From the above reasoning we can find that λ must be at most $-\frac{1}{27}$ or else the inequality fails for the polynomial $f(x)=x^2(x-1)$ at $x=\frac{1}{2}$.

Equality occurs either when $\alpha = \beta = \gamma$ and $x = 0$; or when $\alpha = \beta = 0$, γ is any non negative real, and $x=\frac{\gamma}{2}$.

6. The given function relation can be written as

$$f(x)+f\left(\frac{1}{1-x}\right)=\frac{2}{x}+\frac{2}{x-1} \quad \ldots(1)$$

Let t be any real number other than 0 and 1. Put $x=\frac{1}{1-t}$ in (1); then $\frac{1}{1-x}=1-\frac{1}{t}$ and we have

$$f\left(\frac{1}{1-t}\right)+f\left(1-\frac{1}{t}\right)=-2t+\frac{2}{t} \quad \ldots(2)$$

When $x = 1-\frac{1}{t}$, $\frac{1}{1-x}=t$ and now (1) yields

$$f\left(1-\frac{1}{t}\right)+f(t)=\frac{2t}{t-1}-2t \quad \ldots(3)$$

Also replacing x by t in (1), we get

$$f(t)+f\left(\frac{1}{1-t}\right)=\frac{2}{t}+\frac{2}{t-1} \quad ..(4)$$

Now adding (3) and (4) and subtracting (2), we obtain

$$f(t)=\frac{t+1}{t-1}$$

7. In the binomial expansion of $\left(1+\frac{6}{13}\right)^{15}$, the third term $\frac{15 \cdot 14}{1 \cdot 2}\left(\frac{6}{13}\right)^2 > 13.$

Therefore $\left(1+\frac{6}{13}\right)^{15} > 13 \Rightarrow \left(1+\frac{6}{13}\right)^{31} > 13^2$

$\Rightarrow 19^{31} > 13^{33} \Rightarrow 19^{93} > 13^{99}$

In the binomial expansion of $(1 + 18)^{93}$ the first term is 1; the second term is $93.\ 18 \equiv 12 \cdot 18 \pmod{81}$; all other terms are divisible by 81. In the expansion of

$(1 + 12)^{99}$ the first term is 1; the second term is 99. 12 ≡ 18. 12 (mod 81); the next two terms $\frac{99\cdot 98}{1\cdot 2}\cdot 12^2, \frac{99\cdot 98\cdot 97}{1\cdot 2\cdot 3}12^3$ are divisible by 81; the other terms being multiples of 12^4 are also divisible by 81. Hence we have $19^{93} \equiv 13^{99}(\text{mod } 81)$.

$\therefore\ 162 \mid 19^{93} - 13^{99}$ ($\because\ 19^{93} - 13^{99}$ is even)

8. Let x (resp. y) be the number of days on which 9 (resp. 10) soldiers do guard duty. Let k be the number of days on which every soldier gets duty. Then

$9x + 10y = 33k$ ...(1)

So we have to minimize $x + y$ subject to (1). Obviously when $k = 1$, (1) has no non - negative integral solutions for x, y. So $k \geq 2$, then $10(x + y) \geq 9x + 10y = 33k \geq 66 \Rightarrow x + y \geq 7x + y$ attains this minimum when $k = 2$; in that case, $y = 3$, and $x = 4$.

Here is one way of allocating the duty to the soldiers.

Let $D_1 = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$D_2 = \{10, 11, 12, 13, 14, 15, 16, 17, 18\}$,

$D_3 = \{19, 20, 21, 22, 23, 24, 25, 26, 27\}$,

$D_4 = \{28, 29, 30, 31, 32, 33, 1, 2, 3\}$,

$D_5 = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$,

$D_6 = \{14, 15, 16, 17, 18, 19, 20, 21, 22, 23\}$

and $D_7 = \{24, 25, 26, 27, 28, 29, 30, 31, 32, 33\}$

For $k = 1,..., 7$, and $j = 1,...,33$, the soldier S_j assigned duty on k^{th} day if $j \in D_k$.

9. For $p = 2$, the given quotient is not an even integer and so we can assume p is an odd prime. Then by Fermat's Little theorem $2^{p-1} - 1$ is divisible by p. Suppose that for some integer a, $2^{p-1} - 1 = pa^2$. Since p is odd we have that

$$\left(2^{\frac{(p-1)}{2}} - 1\right)\left(2^{\frac{(p-1)}{2}} + 1\right) = pa^2$$

Both the factors on the L.H.S. are odd and hence they are relatively prime to each other which implies that p divides exactly one of the two factors. Therefore either

$$2^{\frac{(p-1)}{2}} - 1 = px^2, 2^{\frac{(p-1)}{2}} + 1 = y^2$$

or $2^{\frac{(p-1)}{2}} - 1 = x^2, 2^{\frac{(p-1)}{2}} + 1 = py^2$

Case – I: Suppose $2^{\frac{(p-1)}{2}} - 1 = px^2, 2^{\frac{(p-1)}{2}} + 1 = y^2$.

But then $2^{\frac{(p-1)}{2}} = (y-1)(y+1)$ for which the only solution is $y = 3$ *i.e.*, $p = 7$. When $p = 7$ observe that the given quotient is indeed a square.

Case – II: Suppose $2^{\frac{(p-1)}{2}} - 1 = x^2, 2^{\frac{(p-1)}{2}} + 1 = py^2$.

Then if $p > 3$ we get that x^2 leaves remainder 3 when divided by 4 which is impossible. Thus p can only be equal to 3; when $p = 3$ also we observe that the given quotient is indeed a square.

Thus the primes p for which the quotient $\frac{(2^{p-1} - 1)}{p}$ is a square are $p = 3, 7$.

10. Consider 2001 numbers $\frac{1}{k}$, $1001 \leq k \leq 3001$

Using AM-HM inequality, we get

$$\left(\sum_{k=1001}^{3001} k\right)\left(\sum_{k=1001}^{3001} \frac{1}{k}\right) > (2001)^2$$

But $\sum_{k=1001}^{3001} k = (2001)^2$

Hence we get the inequality

$$\sum_{k=1001}^{3001} \frac{1}{k} > 1$$

On the other hand, grouping 500 terms at a time, we also have

$$S = \sum_{k=1001}^{3001} \frac{1}{k}$$

$$< \frac{500}{1000} + \frac{500}{1500} + \frac{500}{2000} + \frac{500}{2500} + \frac{1}{3001}$$

$$< \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{3000}$$

$$= \frac{3851}{3000} < \frac{4}{3}$$

Note: We can sharpen the above inequality. Consider the sum

$$S = \sum_{k=n+1}^{3n+1} \frac{1}{k}$$

There are $2n + 1$ terms in the sum and the middle term is $\frac{1}{2n+1}$. We can write the sum in the form

$$S = \frac{1}{2n+1} + \sum_{k=1}^{n}\left(\frac{1}{2n+1+k} + \frac{1}{2n+1-k}\right)$$

$$= \frac{1}{2n+1} + \frac{2}{(2n+1)} + \sum_{k=1}^{n} \frac{1}{1-\left(\frac{k}{2n+1}\right)^2}$$

For $0 < a < \frac{1}{2}$, we have

$$1 + a < \frac{1}{1-a} < 1 + 2a$$

Thus we get the bounds

$$\frac{1}{2n+1} + \frac{2}{2n+1}\sum_{k=1}^{n}\left[1+\left(\frac{k}{2n+1}\right)^2\right] < S$$

and $S < \frac{1}{2n+1} + \frac{2}{2n+1}\sum_{k=1}^{n}\left[1+2\left(\frac{k}{2n+1}\right)^2\right]$

This on simplification gives

$$1 + \frac{2}{(2n+1)^3}\sum_{k=1}^{n} k^2 < S < 1 + \frac{4}{(2n+1)^3}\sum_{k=1}^{n} k^2$$

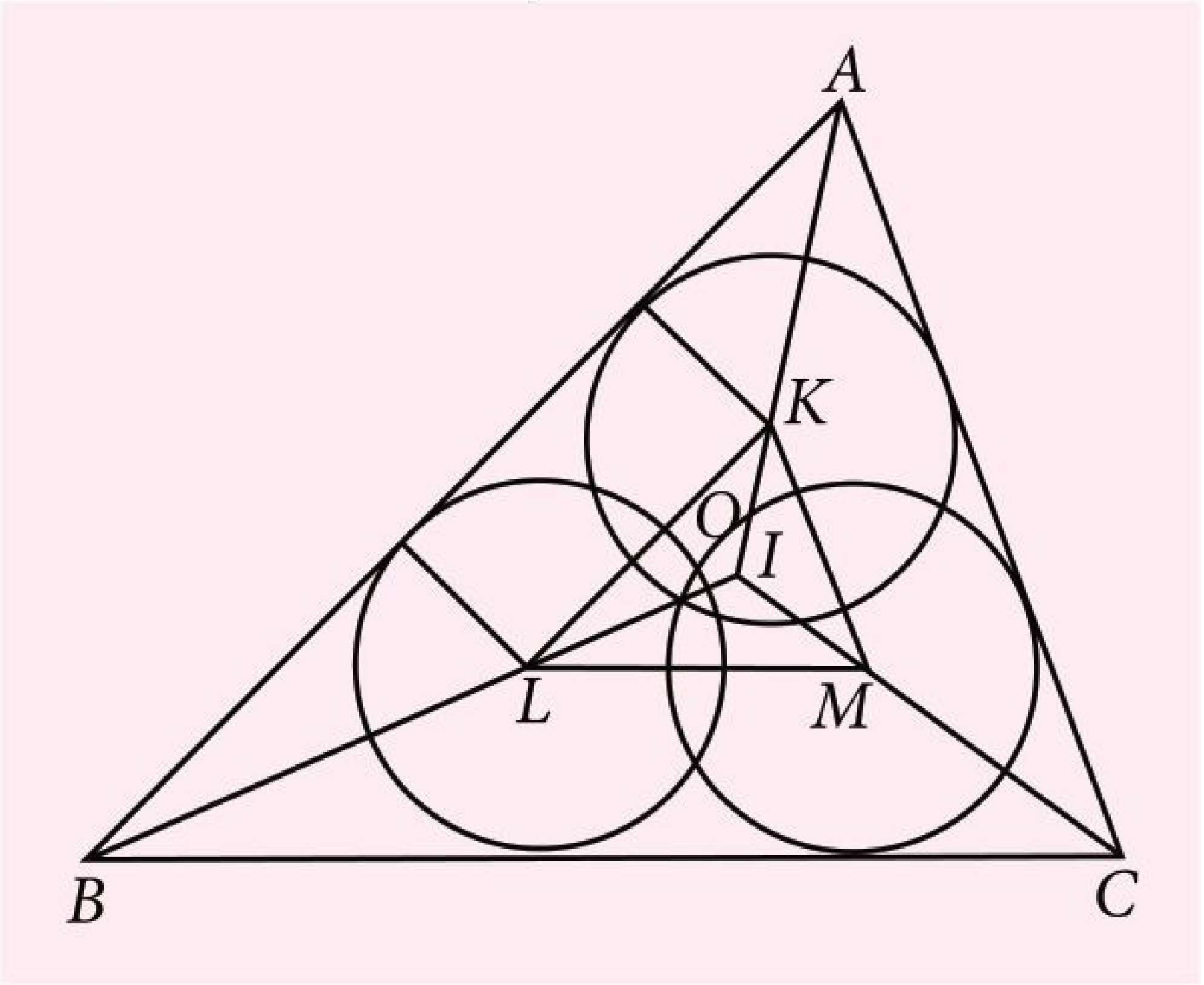
Now using the identity $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, the inequality simplifies to

$$1 + \frac{n(n+1)}{3(2n+1)^2} < S < 1 + \frac{2}{3}\frac{n(n+1)}{(2n+1)^2}$$

But for $n \geq 1$, we also have $\frac{2}{9} \leq \frac{n(n+1)}{(2n+1)^2} \leq \frac{1}{4}$

This leads to $\frac{29}{27} < S < \frac{7}{6}$

11. Let K, L, M be the centres of the three circles of equal radii, meeting in a common point O, and pairwise touching some side of the triangle ABC in which they lie. Since the three circles have equal radii, we see that O is equidistant from the points K, L, M and so is the circumcentre of the triangle KLM. Also for same reason, the sides of the triangle KLM are parallel to the corresponding sides of the triangle ABC. (For instance, KL is parallel to AB, as K and L are equidistant from AB.)



Further AK, BL, CM are the bisectors of angles A, B, C of triangle ABC. So not only they meet in I, the incentre of the triangle ABC but also KI, LI, MI are the bisectors of the angles of triangle KLM, implying that I is also the incentre of triangle KLM.

It follows, from the relation $\frac{IK}{IA} = \frac{IL}{IB} = \frac{IM}{IC}$ that triangle KLM is homothetic to triangle ABC with respect to I, the centre of homothety. (This simply means that triangle ABC is a dilation (or an enlargement) of triangle KLM as seen from I).

A property of homothety is that the centre of homothety and any two corresponding points of the homothetic figures are collinear. Here, in particular, I and the circumcentres of triangles KLM and ABC have to be collinear. This is precisely what was to be proved.

12. We shall look at the problem from a general viewpoint. For any positive integer n, let T_n denote the number of permutations of $(P_1, P_2, P_3, \ldots, P_n)$ of 1, 2, 3, ..., n such that for each k, $1 \leq k \leq n$, $(P_1, P_2, P_3, \ldots, P_n)$ is not a permutation of 1, 2, ..., k. We shall obtain a formula for T_n which expresses T_n in terms of $T_1, T_2, \ldots, T_{n-1}$ $(n > 1)$. (Such a relation is called a recurrence relation for T_n.)

Consider any permutation $(P_1, P_2, P_3, \ldots, P_n)$ of 1, 2, ..., n. There is always a least positive integer k such that $(P_1, P_2, P_3, \ldots P_k)$ is a permutation of 1, 2, ..., k. In fact k may be any integer in the set $\{1, 2, \ldots, n\}$; and those permutations for which $k = n$ are $(1, 2, \ldots, n)$ for all of which k is the least positive integer satisfying the above property is $T_k \cdot (n - k)!$, by our definition of T_n. The second factor corresponds to the permutations of $k + 1$, $k + 2$, ..., n which fill up the remaining $(n - k)$ places. Since there are $n!$ permutations in all, we obtain

$$n! = \sum_{k=1}^{n} T_k \cdot (n-k)!$$
$$= T_1 \cdot (n-1)! + T_2 \cdot (n-2)! + \ldots\ldots + T_{n-1} \cdot 1! + T_n \cdot 0!$$

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1.	(a)	**2.**	(a)	**3.**	(b)	**4.**	(d)	**5.**	(d)
6.	(b)	**7.**	(c)	**8.**	(c)	**9 .**	(a,b)	**10.**	(a,c)
11.	(a,c,d)	**12.**	(c)	**13.**	(b)	**14.**	(b)	**15.**	(d)
16.	(c)	**17.**	(4)	**18.**	(6)	**19.**	(5)	**20.**	(3)

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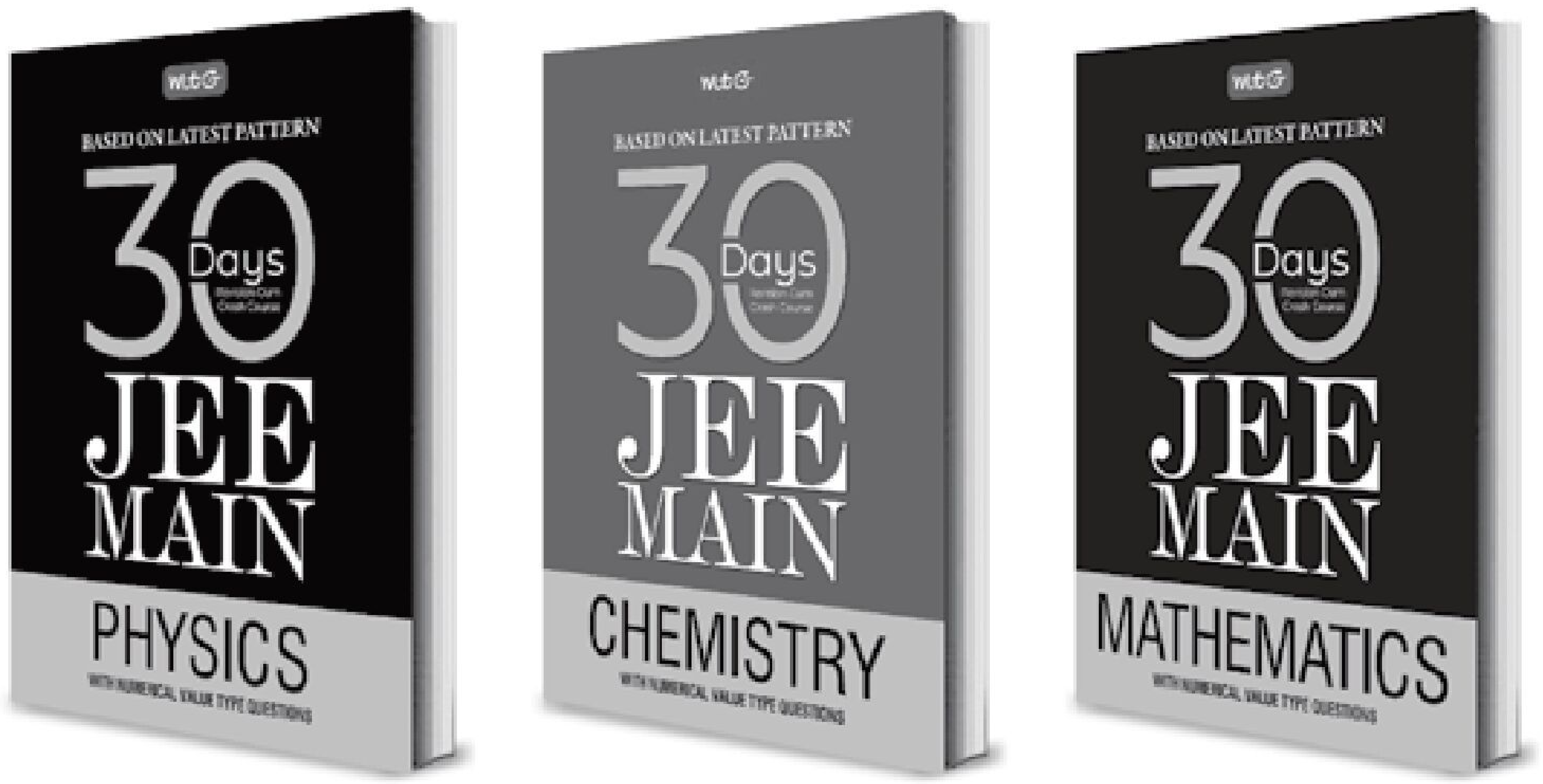
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Hence

$T_n = n! - T_1.(n-1)! - T_2.(n-2)! \,.... - T_{n-1}.1!$

Clearly $T_1 = 1$

$T_2 = 2! - T_1 \,.\, 1! = 2 - 1 = 1$

$T_3 = 3! - T_1 \,.\, 1! - T_2 \,.\, 1! = 6 - 2 - 1 = 3$

$T_4 = 4! - T_1 \,.\, 1! - T_2 \,.\, 2! - T_3 \,.\, 1! = 24 - 6 - 2 - 3 = 13.$

$$T_4 = 5! - T_1 \,.\, 4! - T_2 \,.\, 3! - T_3 \,.\, 2! - T_4 \,.\, 1!$$
$$= 120 - 24 - 6 - 6 - 13 = 71$$

$$T_6 = 6! - T_1 \,.\, 5! - T_2 \,.\, 4! - T_3 \,.\, 3! - T_4 \,.\, 2! - T_5 \,.\, 1!$$
$$= 720 - 120 - 24 - 18 - 26 - 71 = 461.$$

Thus the required number is 461.

13. 1st Solution:

i) If $f : A \to A$ is a bijective function then there is a unique bijective function $g : A \to A$ such that $fog = gof = I_A$, then identity function on A. The function g is called the inverse of f and is denoted by f^{-1}. Thus, $fof^{-1} = I_A = f^{-1}of$

ii) $fo\, I_A = f = I_A\, of$

iii) If f and g are bijections from A to A, then so are gof and fog.

iv) If f, g, h are bijective functions from A to A and $fog = foh$, then $g = h$.

Apply f^{-1} at left to both sides to obtain $g = h$.

Coming to the problem, since A has n elements, we see that there are only finitely many (in fact, $n!$) bijective functions from A to A as each bijective function f gives a permutation of $\{1, 2, 3, ..., n\}$ by taking $\{f(1), f(2), ..., f(n)\}$. Since f is a bijective function from A to A, so is each of the function in the sequence:

$$fof = f^2,\ fofof = f^3,\ ....f^n,\ ......$$

All these cannot be distinct, Since there are only finitely many bijective functions from A to A. Hence for some two distinct positive integers m and n, $m > n$ say, we must have $f^m = f^n$

If $n = 1$, we take $M = m$, to obtain the result. If $n > 1$, multiply both sides by $(f^{-1})^{n-1}$, to get $f^{m-n+1} = f$. We take $M = m - n + 1$ to get the relation

$$f^M = f,\ (M > 1)$$

Note : this means

$$f^M(i) = f(i) \text{ for all } i \in A.$$

2nd Solution: Take any element r in the set A and consider the sequence of elements

$$r, f(r), (fof)(r), (fofof)(r), ...$$

obtained by applying f successively. Since A has only n elements there must be repetitions in the above sequence. But when the first repetition occurs, this must be r itself; for, if the above sequence looks (for instance) like

$$r, a, b, c, d, e, c, ...$$

where the first repetition is an element c other than r, this would imply

$$f(b) = c \text{ and } f(e) = c,$$

contradicting the fact that f is a bijection. Thus for some positive integer $l_r \geq 1$, we have $f^{l_r}(r) = r$

This is true for each r in the set $A = 1, 2, ..., n$. By taking M to be the l.c.m. of $l_1, l_2, ..., l_r$ we get

$$f^M(r) = r \text{ for each } r \in A.$$

[Note: If f itself is the identity function the above proof fails because each $l_r = 1$. But in this case we may take M to be any integer greater than or equal to 2].

◆ ◆

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

			2	4		5							6		5		9			
	8			9		4							3					8		1
		3											7						6	
	5	7			6		3	8							6			3	2	
1	4											1			9					
												8		3					1	
5			9			7				1	9		4						9	
			7	8				5					8		4	1		6		2
2		8	1				6					2								
						9	1					7								
									8	3										
	8		7			2	3						5	1	4		3			
9		5	8						5			9	2					1		
2	6							8	6						6					
1			2				8						4	8	2					
	4							5						2		9		3		
6		2											6			7				
7						9	5						3		8				6	
			1	5	3														4	2
				6								1	8			6				

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

YOU ASK
WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If z_1, z_2, z_3 are complex numbers such that

$$|z_1|=|z_2|=|z_3|=1 \text{and } \frac{z_1^2}{z_2z_3}+\frac{z_2^2}{z_3z_1}+\frac{z_3^2}{z_1z_2}=-1,$$

then find the value of $|z_1+z_2+z_3|$

(Abhijit Kr. Bhubaneshwar)

Ans. Let $z = z_1 + z_2 + z_3$. Then

$$\bar{z}=\bar{z}_1+\bar{z}_2+\bar{z}_3=\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}$$

$$=\frac{z_1z_2+z_2z_3+z_3z_1}{z_1z_2z_3}$$

$\Rightarrow\ \bar{z}b=z_1z_2+z_2z_3+z_3z_1$, where $b = z_1z_2z_3$.

Now,

$$\frac{z_1^2}{z_2z_3}+\frac{z_2^2}{z_3z_1}+\frac{z_3^2}{z_1z_2}=-1\Rightarrow z_1^3+z_2^3+z_3^3=-z_1z_2z_3$$

$\Rightarrow\ z_1^3+z_2^3+z_3^3-3z_1z_2z_3=-4b$

$\Rightarrow\ (z_1+z_2+z_3)[(z_1+z_2+z_3)^2-3(z_1z_2+z_2z_3+z_3z_1)]$
$= -4b$

$\Rightarrow\ z(z^2-3\bar{z}b)=-4b$

$\Rightarrow\ z^3 - 3|z|^2 b + 4b = 0$

$\Rightarrow\ z^3 = (3|z|^2 - 4)b$

$\Rightarrow\ |z|^3 = |3|z|^2 - 4|$ (since $|b| = |z_1z_2z_3| = 1$)

Case I : Suppose that $3|z|^2 \geq 4$. Then

$|z|^3 = 3|z|^2 - 4$

$\Rightarrow\ |z|^3 - 3|z|^2 + 4 = 0$

$\Rightarrow\ (|z| - 2)(|z|^2 - |z| - 2) = 0$

$\Rightarrow\ (|z| - 2)(|z| - 2)(|z| + 1) = 0$

$\Rightarrow\ |z| = 2$

Case II : Suppose that $3|z|^2 < 4$. Then

$|z|^3 = |3|z|^2 - 4| = 4 - 3|z|^2$

$\Rightarrow\ |z|^3 + 3|z|^2 - 4 = 0$

$\Rightarrow\ (|z| - 1)(|z|^2 + 4|z| + 4) = 0$

$\Rightarrow\ (|z| - 1)(|z| + 2)^2 = 0 \Rightarrow |z| = 1.$

2. Find the general solution of the equation $\sin^4 2x + \cos^4 2x = \sin 2x \cos 2x$. ***(Anand W.B.)***

Ans. We have,

$$(\sin^2 2x + \cos^2 2x)^2 - 2\sin^2 2x \cos^2 2x = \sin 2x \cos 2x$$

$\Rightarrow\ 2 \sin^2 2x \cos^2 2x + \sin 2x \cos 2x - 1 = 0$

Put $\sin 2x \cos 2x = t$. Therefore, we have

$2t^2 + t - 1 = 0$

$\Rightarrow\ (2t - 1)(t + 1) = 0 \Rightarrow\ t=\frac{1}{2},-1$

Case I : When $t = -1$, we have

$\sin 2x \cos 2x = -1$

$\Rightarrow\ \sin 4x = -2$, which is impossible.

Therefore $t = -1$ is to be rejected.

Case II : When $t = 1/2$, we have

$\sin 4x = 1$

$\Rightarrow\ 4x = 2n\pi + (\pi/2)$

$\Rightarrow\ x=\frac{n\pi}{2}+\frac{\pi}{8}=(4n+1)\frac{\pi}{8}$

3. A person has to go through three successive tests. The probability of his passing first test is p. If he fails in one of the tests, then the probability of his passing next test is $p/2$, otherwise it remains the same. For selection, the person must pass atleast two tests. Find the probability that the person has to be selected.

(Devika Kochi)

Ans. Let E_i $(i = 1, 2, 3)$ be the event of the person passing the i^{th} test and E is the event that he is selected. Then

$$E=(E_1\cap E_2)\cup(E_1\cap\bar{E}_2\cap E_3)\cup(\bar{E}_1\cap E_2\cap E_3)$$

Therefore

$$P(E)=P(E_1)P(E_2\,|\,E_1)+P(E_1)P(\bar{E}_2\,|\,E_1)P(E_3\,|\,\bar{E}_2)$$
$$+P(\bar{E}_1)P(E_2\,|\,\bar{E}_1)P(E_3\,|\,E_2)$$

$=p\cdot p+p(1-p)\frac{p}{2}+(1-p)\frac{p}{2}\cdot p$

$= 2p^2 - p^3.$

4. If $\vec{x}\times\vec{a}+(\vec{x}\cdot\vec{b})\vec{c}=\vec{d}$, then prove that

$\vec{x}=\lambda\vec{a}+\vec{a}\times\frac{\vec{a}\times(\vec{d}\times\vec{c})}{(\vec{a}\cdot\vec{c})\vec{a}^2}$, where λ is a scalar.

(Rajkumar Patna)

Ans. Here $\vec{x}\times\vec{a}+(\vec{x}\cdot\vec{b})\vec{c}=\vec{d}$

$\therefore\ \{\vec{x}\times\vec{a}+(\vec{x}\cdot\vec{b})\vec{c}\}\times\vec{c}=\vec{d}\times\vec{c}$

$\Rightarrow\ (\vec{x}\times\vec{a})\times\vec{c}=\vec{d}\times\vec{c}$ $\quad[\because\ \vec{c}\times\vec{c}=\vec{0}]$

$$\Rightarrow\quad (\vec{x}\cdot\vec{c})\vec{a}-(\vec{a}\cdot\vec{c})\vec{x}=\vec{d}\times\vec{c}$$

$$\therefore\quad \vec{a}\times\{(\vec{x}\cdot\vec{c})\vec{a}-(\vec{a}\cdot\vec{c})\vec{x}\}=\vec{a}\times(\vec{d}\times\vec{c})$$

$$\Rightarrow\quad -(\vec{a}\cdot\vec{c})(\vec{a}\times\vec{x})=\vec{a}\times(\vec{d}\times\vec{c})\quad \{\because\ \vec{a}\times\vec{a}=\vec{0}\}$$

$$\Rightarrow\quad \vec{x}\times\vec{a}=\frac{\vec{a}\times(\vec{d}\times\vec{c})}{\vec{a}\cdot\vec{c}}$$

$$\Rightarrow\quad \vec{a}\times(\vec{x}\times\vec{a})=\vec{a}\times\frac{\vec{a}\times(\vec{d}\times\vec{c})}{\vec{a}\cdot\vec{c}}$$

$$\Rightarrow\quad (\vec{a}\cdot\vec{a})\vec{x}-(\vec{a}\cdot\vec{x})\vec{a}=\vec{a}\times\frac{\vec{a}\times(\vec{d}\times\vec{c})}{\vec{a}\cdot\vec{c}}$$

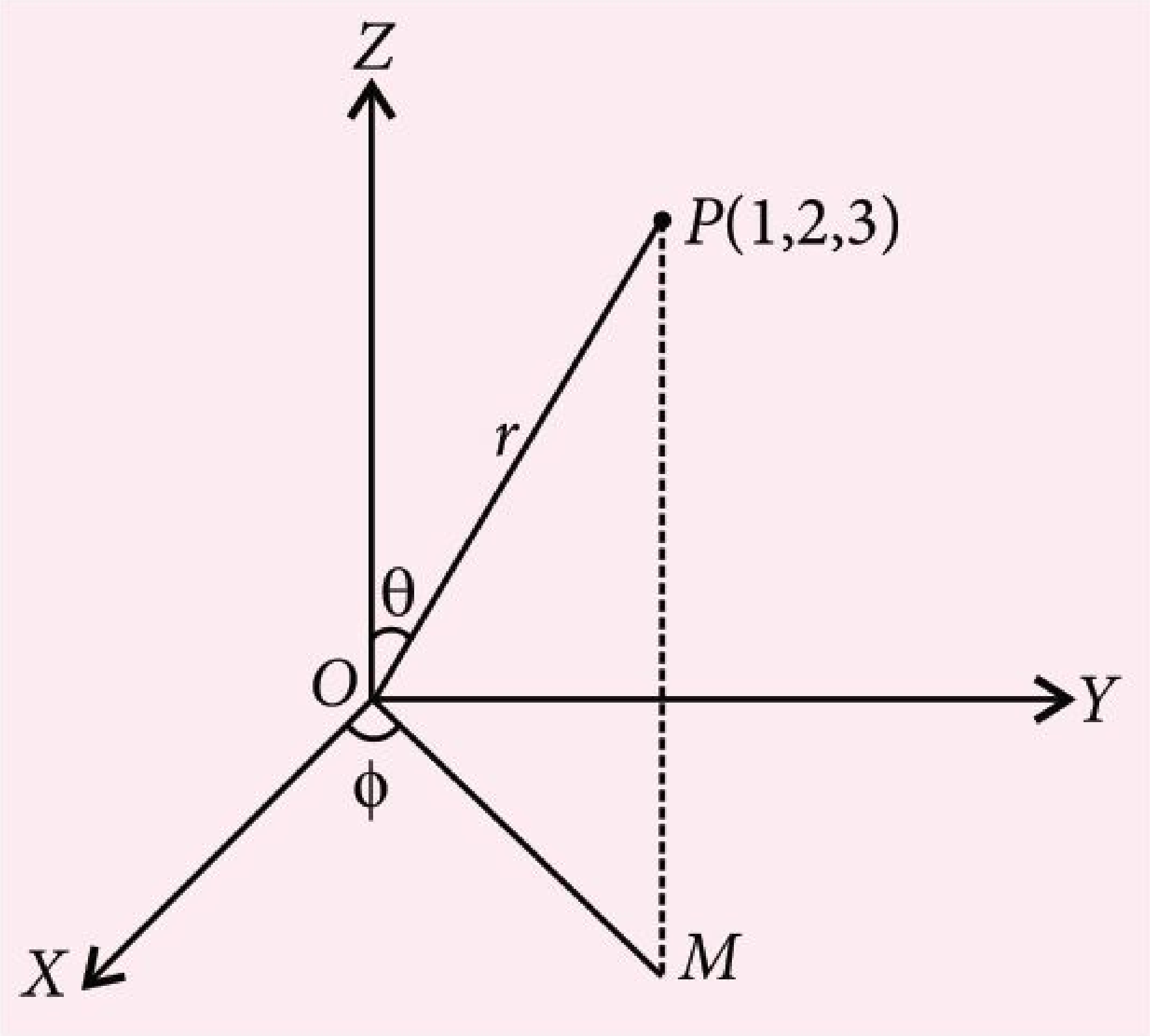
$$\Rightarrow\quad \vec{a}^2\vec{x}=(\vec{a}\cdot\vec{x})\vec{a}+\vec{a}\times\frac{\vec{a}\times(\vec{d}\times\vec{c})}{\vec{a}\cdot\vec{c}}$$

$$\therefore\quad \vec{x}=\frac{(\vec{a}\cdot\vec{x})}{\vec{a}^2}\vec{a}+\vec{a}\times\frac{\vec{a}\times(\vec{d}\times\vec{c})}{(\vec{a}\cdot\vec{c})\vec{a}^2}$$

$$=\lambda\vec{a}+\vec{a}\times\frac{\vec{a}\times(\vec{d}\times\vec{c})}{(\vec{a}\cdot\vec{c})\vec{a}^2},\ \text{where } \lambda=\frac{\vec{a}\cdot\vec{x}}{\vec{a}^2}=\text{scalar.}$$

5. Let PM be the perpendicular from the point $P(1, 2, 3)$ to the xy plane. If OP makes an angle θ with the positive direction of the z-axis and OM makes an angle ϕ with the positive direction of the x-axis, where O is the origin, then find θ and ϕ.

(Sahil U.P.)



Ans. We know that if $P = (x, y, z)$ then

$$x = r\sin\theta\cdot\cos\phi,\ y = r\sin\theta\cdot\sin\phi,\ z = r\cos\theta$$

$$\therefore\quad 1 = r\sin\theta\cdot\cos\phi,\ 2 = r\sin\theta\cdot\sin\phi,\ 3 = r\cos\theta$$

$$\Rightarrow\quad 1^2 + 2^2 = r^2\sin^2\theta\cdot\cos^2\phi + r^2\sin^2\theta\cdot\sin^2\phi$$

$$= r^2\sin^2\theta(\cos^2\phi + \sin^2\phi) = r^2\sin^2\theta$$

$$\Rightarrow\quad 5 = r^2\sin^2\theta\quad\therefore\quad r\sin\theta = \sqrt{5}$$

(clearly, θ and ϕ are acute)

$$\therefore\quad \frac{r\sin\theta}{r\cos\theta}=\frac{\sqrt{5}}{3}\quad\text{or}\quad \tan\theta=\frac{\sqrt{5}}{3}$$

$$\therefore\quad \theta=\tan^{-1}\left(\frac{\sqrt{5}}{3}\right)$$

$$\text{Also, }\ \frac{1}{2}=\frac{r\sin\theta\cdot\cos\phi}{r\sin\theta\cdot\sin\phi}\ \Rightarrow\ \cot\phi=\frac{1}{2}\ \Rightarrow\ \tan\phi=2.$$

$$\therefore\quad \phi=\tan^{-1}2.$$

6. Find the value of $\displaystyle\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin 2x}dx$ ***(Vikas M.P.)***

Ans. Let $\displaystyle I=\int_0^{\pi/4}\frac{d(\sin x-\cos x)}{9-16(1-\sin 2x)+16}$

$$=\int_0^{\pi/4}\frac{d(\sin x-\cos x)}{25-16(\sin x-\cos x)^2}$$

$$=\frac{1}{16}\int_0^{\pi/4}\frac{d(\sin x-\cos x)}{\left(\frac{5}{4}\right)^2-(\sin x-\cos x)^2}$$

Put $\sin x - \cos x = z$; then

$$x=0\ \Rightarrow\ z=-1;\ x=\frac{\pi}{4}\ \Rightarrow\ z=0$$

$$\therefore\quad I=\frac{1}{16}\int_{-1}^{0}\frac{dz}{\left(\frac{5}{4}\right)^2-z^2}=\frac{1}{16}\cdot\left[\frac{1}{2\cdot\frac{5}{4}}\log\frac{\frac{5}{4}+z}{\frac{5}{4}-z}\right]_{-1}^{0}$$

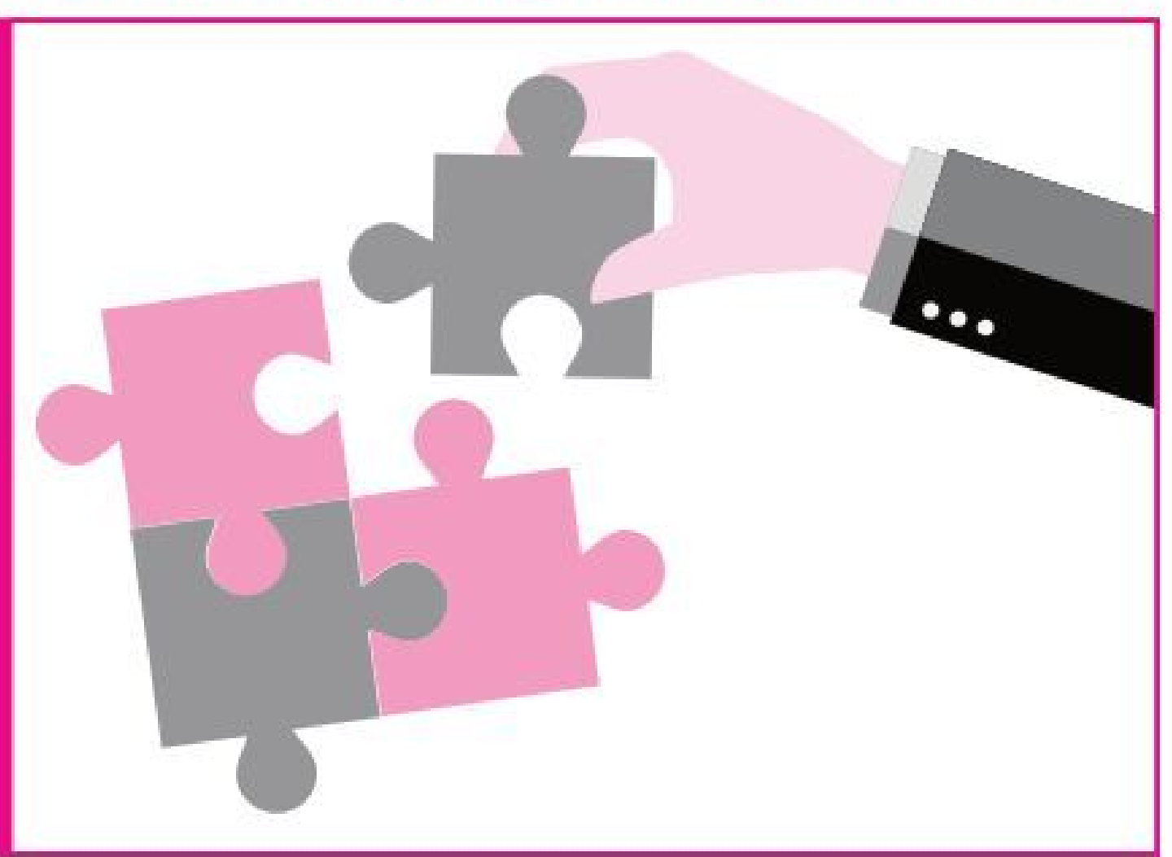
$$=\frac{1}{40}\left[\log\frac{5+4z}{5-4z}\right]_{-1}^{0}=\frac{1}{40}\left\{\log 1-\log\frac{1}{9}\right\}$$

$$=\frac{1}{40}\log 9=\frac{1}{20}\log 3.$$

◆◆

PUZZLE CORNER

ANSWER · JUNE 2021



4– 2	24× 6	1	4	15× 3	5
6	12× 2	3	20× 1	5	4
1– 4	3	2	11+ 5	6	3÷ 1
1– 5	4	120× 6	2÷ 2	1	3
3÷ 1	5	4	36× 3	2	6
3	1	11+ 5	6	2– 4	2

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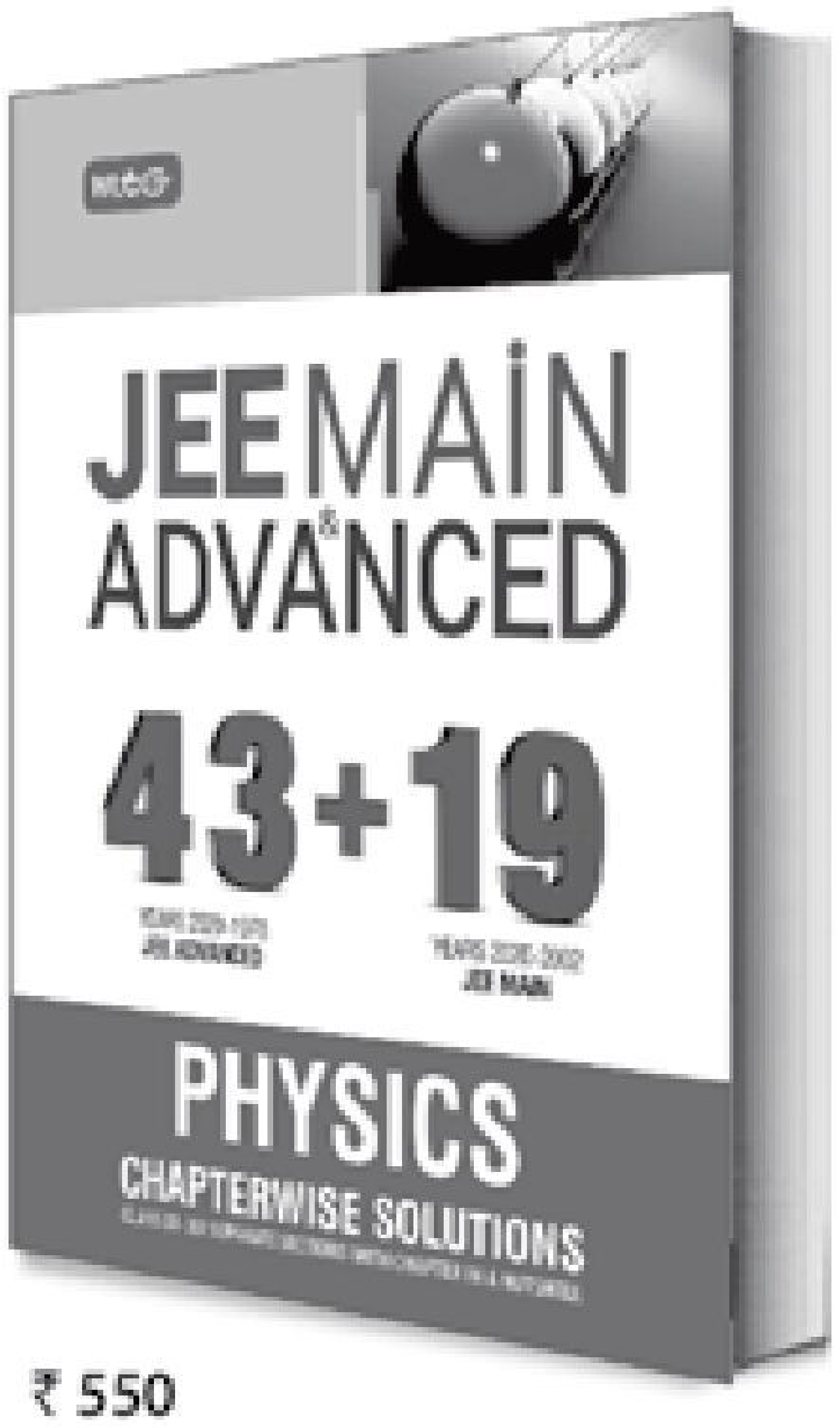
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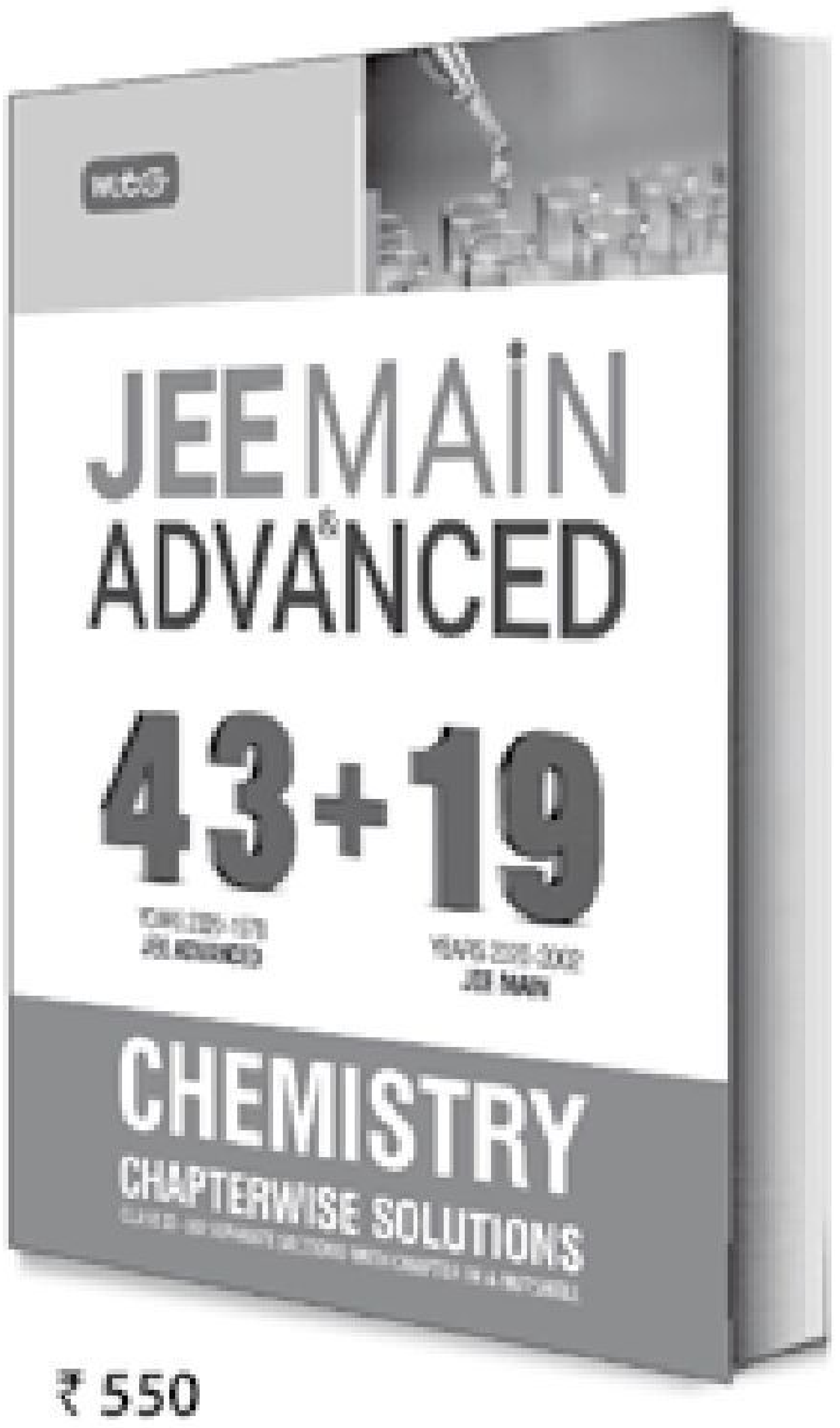
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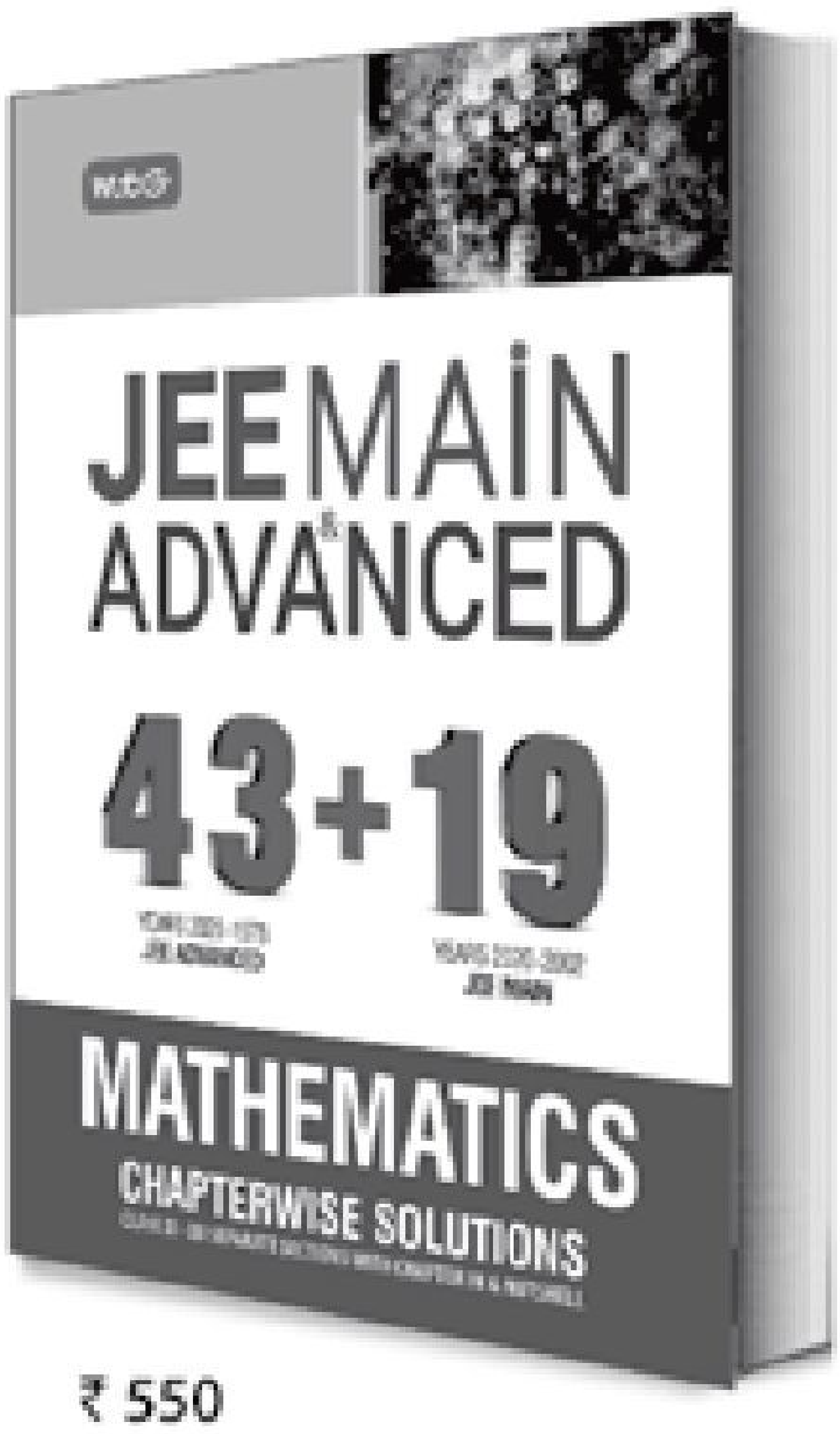
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QUANTITATIVE APTITUDE

Useful for Bank PO, Specialist Officers & Clerical Cadre, BCA, MAT, CSAT, CDS and other such examinations

1. A person has five iron rods with lengths 16, 24, 48, 72, 104 cm each. He wants to convert into pieces of equal length from each of five rods. The least number of total pieces, if there is no wastage of material is

(a) 14 (b) 8 (c) 33 (d) 44

2. The average marks of 30 students are 45 but after checking there are two mistakes found. After adjustment if a student got 45 more marks and other student got 15 less mark then what will be the correct average?

(a) 47 (b) 46 (c) 45 (d) 44

3. By selling 8 dozen pencils, a shopkeeper gains the selling price of 1 dozen, what is gain %?

(a) $14\frac{2}{7}\%$ (b) $13\frac{1}{7}\%$ (c) $87\frac{1}{2}\%$ (d) $12\frac{1}{2}\%$

4. A man, a woman, a boy worked for a contractor for the same time. A man is twice efficient as a woman and a woman is thrice efficient as a boy. ₹ 12500 are given to all of them. What is the sum of money got by a man and a boy together?

(a) ₹ 7500 (b) ₹ 3750 (c) ₹ 8750 (d) ₹ 5000

5. What can be said about the expression of $2^{12n} - 6^{4n}$, where n is a positive integer?

(a) Last digit is 4 (b) Last digit is 8
(c) Last two digits are zero
(d) Last digit is 2

6. The value of $\frac{(0.92)^3 - (0.1)^3}{(0.92)^2 + .092 + (0.01)}$ is

(a) $0.\overline{82}$ (b) 1.02 (c) 0.82 (d) 0.93

7. Two pipes A and B can fill a cistern in 12 hours and 15 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom, it took 80 minutes more to fill the cistern. When the cistern is full, in what time will the leakage empty it?

(a) 17 hours (b) 18 hours
(c) 19 hours (d) none of these

8. If a, b, c, d and e are in continued proportion, then $a : e$ equals

(a) $a^4 : b^3$ (b) $a^3 : b^4$ (c) $a^3 : b^3$ (d) $a^4 : b^4$

9. If simple interest on ₹ 2000 increased by ₹ 60 when the rate % increased by 3% p.a., then time equals

(a) 1 year (b) 2 years (c) 3 years (d) 4 years

10. In a rectangular field 12 m wide and 22 m long, we need to pave a path around its borders of uniform width so as to leave an area of 144 sq. m. Width of the path is

(a) 2.5 m (b) 4 m (c) 3 m (d) 2 m

11. $2\frac{1}{2}\text{ of }\frac{3}{4}\times\frac{1}{2}\div\frac{3}{2}+\frac{1}{2}\div\left\{\frac{3}{2}\left(\frac{2}{3}-\frac{1}{2}\text{ of }\frac{2}{3}\right)\right\}=?$

(a) $2\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $1\frac{5}{8}$

12. In a race of 200 m, A beats B by 31 m and C by 18 m, then by how many metres C will defeat B in a 420 metres race?

(a) 20 m (b) 22 m (c) 25 m (d) 30 m

13. The least value of $(4\sec^2\theta + 9\csc^2\theta)$ is equal to

(a) 25 (b) 19 (c) 7 (d) 1

14. An increase of 20% on price of banana enable a man to purchase 4 bananas less for ₹ 40. The price of 15 bananas before increase in price was

(a) ₹ 10 (b) ₹ 15 (c) ₹ 20 (d) ₹ 25

15. Select the missing number from the given options.
500, 484, 459, ?, 374

(a) 418 (b) 423 (c) 432 (d) 384

16. The number of arrangements that can be made out from the letters of the word 'SUPERSTITIOUS' is

(a) $\frac{12!}{(2!)^3}$ (b) $\frac{13!}{3!(2!)^3}$ (c) $\frac{13!}{(2!)^2}$ (d) $\frac{12!}{(2!)^2}$

17. If $\frac{x^2+y^2+z^2-64}{xy-yz-zx}=-2$ and $x+y=3z,$ then $z=$

(a) 1 (b) 2 (c) 3 (d) 4

18. Reena, Reema and Meera take a car on rent. They use the car for 14 h, 16 h and 22 h respectively. What

amount will be paid by Reema if total rent of car is 2080?
(a) ₹ 1440 (b) ₹ 560 (c) ₹ 880 (d) ₹ 640

19. At what time are the hands together coincide between 2 o'clock and 3 o'clock ?

(a) $10\frac{10}{11}$ mins past 2 (b) $10\frac{9}{11}$ mins past 2

(c) $11\frac{10}{11}$ mins past 2 (d) None of these

20. The height of a bucket is 45 cm. The radii of two circular ends are 28 cm and 7 cm respectively. The volume of the bucket is

(a) 38510 cm^3 (b) 36810 cm^3
(c) 48510 cm^3 (d) 48600 cm^3

21. Three digit numbers are formed by using the digits, 0, 1, 3, 5 and 9. A number is chosen randomly among these numbers, what is the probability that selected number has the same (identical) digits?

(a) 1/50 (b) 1/25 (c) 1/100 (d) 2/25

22. The product of two numbers is 17, then sum of reciprocals of their squares is

(a) $\frac{1}{289}$ (b) $\frac{289}{290}$ (c) $\frac{290}{289}$ (d) $\frac{1}{290}$

23. The calendar 2024 will be same as calendar of year

(a) 2030 (b) 2052
(c) 2035 (d) None of these

24. 25 tonnes of steel cost ₹ 125000. What will be the cost of 270 kg steel?

(a) ₹ 1500 (b) ₹ 1350 (c) ₹ 1450 (d) ₹ 1250

25. If $f(x) = \log_4(\log_5(\log_3(12x - x^2 - 32)))$, then find the domain of $f(x)$.

(a) (–8, –4) (b) (–4, 8) (c) (–7, –5) (d) (5, 7)

26. The population of a town is 126800. It increases by 15% in the first year and decreases 20% in the second year. The population of town at the end of 2 years is

(a) 174984 (b) 116656 (c) 135996 (d) 145820

27. The ratio of the ages of Nishtha and Tina is 3 : 4. After five years the new ratio of their ages will be 4 : 5. What is the age of Nishtha and Tina (in years) respectively?

(a) 20, 15 (b) 15, 20 (c) 12, 16 (d) 21, 28

28. A garrison of 'n' men has enough food to last for 30 days. After 10 days, 50 more joined them. If the food now lasted for 16 days, what is the value of n?

(a) 225 (b) 240 (c) 200 (d) 180

29. In covering a distance 40 km, Ram takes $2\frac{1}{2}$ hrs more than Karan. If the speed of Ram be doubles then his time reduces $1\frac{1}{2}$ hrs less than Karan. The speed of Ram equals

(a) 5 km/h (b) 4 km/h
(c) 3 km/h (d) None of these

30. $\left[\left(\sqrt[5]{x^{-3/5}}\right)^{-5/3}\right]^5 =$

(a) x^5 (b) x^{-5} (c) x (d) x^{-1}

SOLUTIONS

1. (c) : Lengths of rods are 16, 24, 48, 72, 104 (cms each)
H.C.F. of (16, 24, 48, 72, 104) = 8

$$\therefore \text{ Least number of total pieces} = \frac{16+24+48+72+104}{8} = 33$$

2. (d) : Correct observation

$$= \frac{\text{Sum of all observations} - \text{more marks} + \text{less marks}}{\text{Number of observations}}$$

$$= \frac{30\times45-45+15}{30} = \frac{30\times44}{30} = 44$$

3. (a) : Gain = selling price of 1 dozen
Let the S.P. of 1 dozen be ₹ x.
∴ S.P. of 8 dozen = ₹ $8x$
Now cost price of 8 dozen pencils
= selling price – profit = $8x - x$ = ₹ $7x$

$$\therefore \text{ Gain } \% = \frac{x}{7x}\times100 = 14\frac{2}{7}\%$$

4. (c) : From problem
Efficiency of a man : woman : boy
2 : 1
3 : 1
∴ 2 × 3 : 3 × 1 : 1 × 1 = 6 : 3 : 1

$$\therefore \text{ Share of a man and boy } = \frac{(6+1)}{10}\times12500$$

= ₹ (7 × 1250) = ₹8750

5. (c) : $2^{12n} - 6^{4n} = (2^{12})^n - (6^4)^n = (4096)^n - (1296)^n$
$= a^n - b^n$ (where $a = 4096$ and $b = 1296$)
$= (a - b)\,(a^{n-1} + b\,a^{n-2} + b^2\,a^{n-3} + b^3a^{n-4} + ... + b^{n-1})$.
$= (a-b)\,(\lambda)$, where $\lambda = (a^{n-1} + b\,a^{n-2} + ... + b^{n-1})$
$= (4096 - 1296)\,\lambda = 2800\,\lambda$
Thus, last two digits are zero.

6. (c)

7. (d) : Part filled by pipe A in 1 hour = 1/12
Part filled by pipe B in 1 hour = 1/15

$$\therefore \text{ Part filled by } (A + B) \text{ in 1 hour } = \frac{1}{12}+\frac{1}{15} = \frac{3}{20}$$

∴ Time taken by these pipes to fill the cistern
= 20/3 = 6 hours 40 minutes

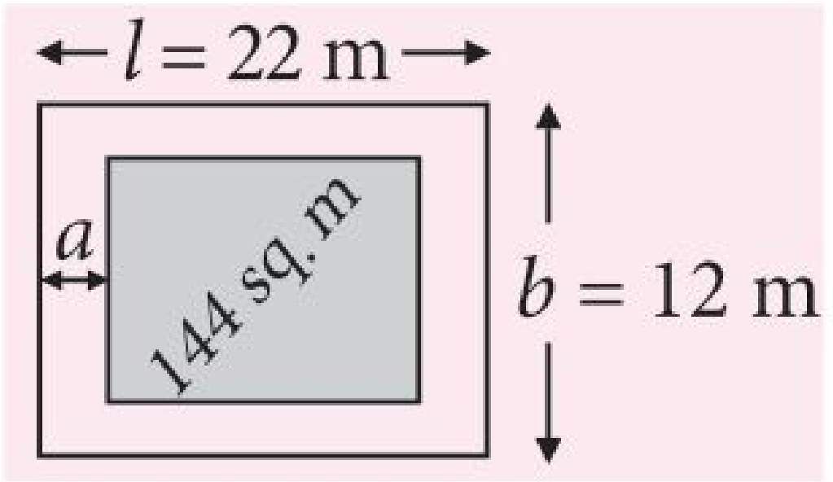
Due to leakage the time taken = 6 hours 40 minutes + 80 minutes = 8 hours

$\therefore$ Part filled in 1 hour = 1/8

$\therefore$ Part emptied by leakage in 1 hour $=\frac{1}{8}-\frac{3}{20}=\frac{5-6}{40}=\frac{-1}{40}$

$\therefore$ Leakage can empty the cistern in 40 hours.

8. (d) **9. (a)**



10. (d) : Let the width of the path be 'a' m. Length of field (outer rectangle) = 22 m

Breadth (width) of field (outer rectangle) = 12 m

$\therefore$ Length of inner rectangle = $(22 - 2a)$ m and Breadth of inner rectangle = $(12 - 2a)$ m

According to problem, $(22 - 2a)(12 - 2a) = 144$

$\Rightarrow (11 - a)(6 - a) = 36 \Rightarrow a^2 - 17a + 30 = 0$

$\Rightarrow (a - 2)(a - 15) = 0$

$\Rightarrow a = 2$ ($\because$ $a = 15$ is not possible)

11. (d)

12. (d) : Distance of race = 200 m

A beats B by 31 m and C by 18 m

$\therefore$ $A : B = 200 : (200 - 31) = 200 : 169$

and $A : C = 200 : (200 - 18) = 200 : 182$

$\therefore$ $C:B=\frac{C}{B}=\frac{C}{A}\times\frac{A}{B}=\frac{182}{200}\times\frac{200}{169}=182:169$

When C runs 182 m, B runs 169 m

When C runs 420 m, then B will run $=\frac{169}{182}\times 420=390$ m

$\therefore$ C will defeat B by $(420 - 390) = 30$ m.

13. (a)

14. (d) : Let the original price of banana be ₹ x.

$\therefore$ New price is 20% more than x $=\frac{120\times x}{100}=$ ₹ $\frac{6x}{5}$

Difference in the number of bananas before and after increase of price = 4

$\therefore$ $\frac{40}{x}-\frac{40\times 5}{6x}=4$

$\Rightarrow$ x (original price of 1 banana) = ₹ (5/3)

$\therefore$ Cost of 15 bananas before increase of price = (15 × 5)/3 = ₹ 25

15. (b)

16. (b) : For the number of arrangements from the word 'SUPERSTITIOUS'

n = 13, S = 3, U = 2, T = 2, I = 2 and other letters are distinct.

$\therefore$ Required arrangements $=\frac{13!}{3!(2!)^3}$

17. (d)

18. (d) : Reena's share : Reema's share : Meera's share = Ratio of time period for which car is used = 14 : 16 : 22 = 7 : 8 : 11

$\therefore$ Sum of ratio's = 7 + 8 + 11 = 26

Total rent = ₹ 2080

$\therefore$ Amount paid by Reema $=\frac{8\times 2080}{26}=8\times 80=$ ₹ 640

19. (a) : Here n = 2

$\therefore$ The time when hands be together

$=\left(5\times n\times\frac{60}{55}\right)$ mins past n o'clock

$=5\times 2\times\frac{60}{55}=\frac{120}{11}=10\frac{10}{11}$ mins past 2

20. (c)

21. (b) : Given digits are 0, 1, 3, 5, 9

$\therefore$ Total possible 3-digit numbers = 4 × 5 × 5 (as repetition is allowed) = 100 out of which only 111, 333, 555, 999 (*i.e.*, 4 numbers) having all digits identical.

$\therefore$ Required probability $=\frac{4}{100}=\frac{1}{25}$.

22. (c)

23. (b) : The year 2024 is a leap year and a leap year in 99% case repeats after 28 years.

$\therefore$ 2024 + 28 = 2052.

24. (b) : $\because$ 1 tonne = 1000 kg

$\therefore$ 25 tonnes = 1000 × 25 = 25000 kg

Now, cost of 25000 kg steel = ₹ 125000 [Given]

Cost of 1 kg steel = $\frac{125000}{25000}$ = ₹ 5

$\therefore$ Cost of 270 kg steel = ₹ (5 × 270) = ₹ 1350

25. (d)

26. (b) : Here r_1 = 15%, r_2 = 20% and P_0 = 126800

$\therefore$ Population at the end of 2 years

$=126800\times\left(1+\frac{15}{100}\right)\left(1-\frac{20}{100}\right)$ = 116656

27. (b)

28. (c) : Here $x = n$, $y = 50$, $a = 30$, $b = 10$, Sufficient days = 16

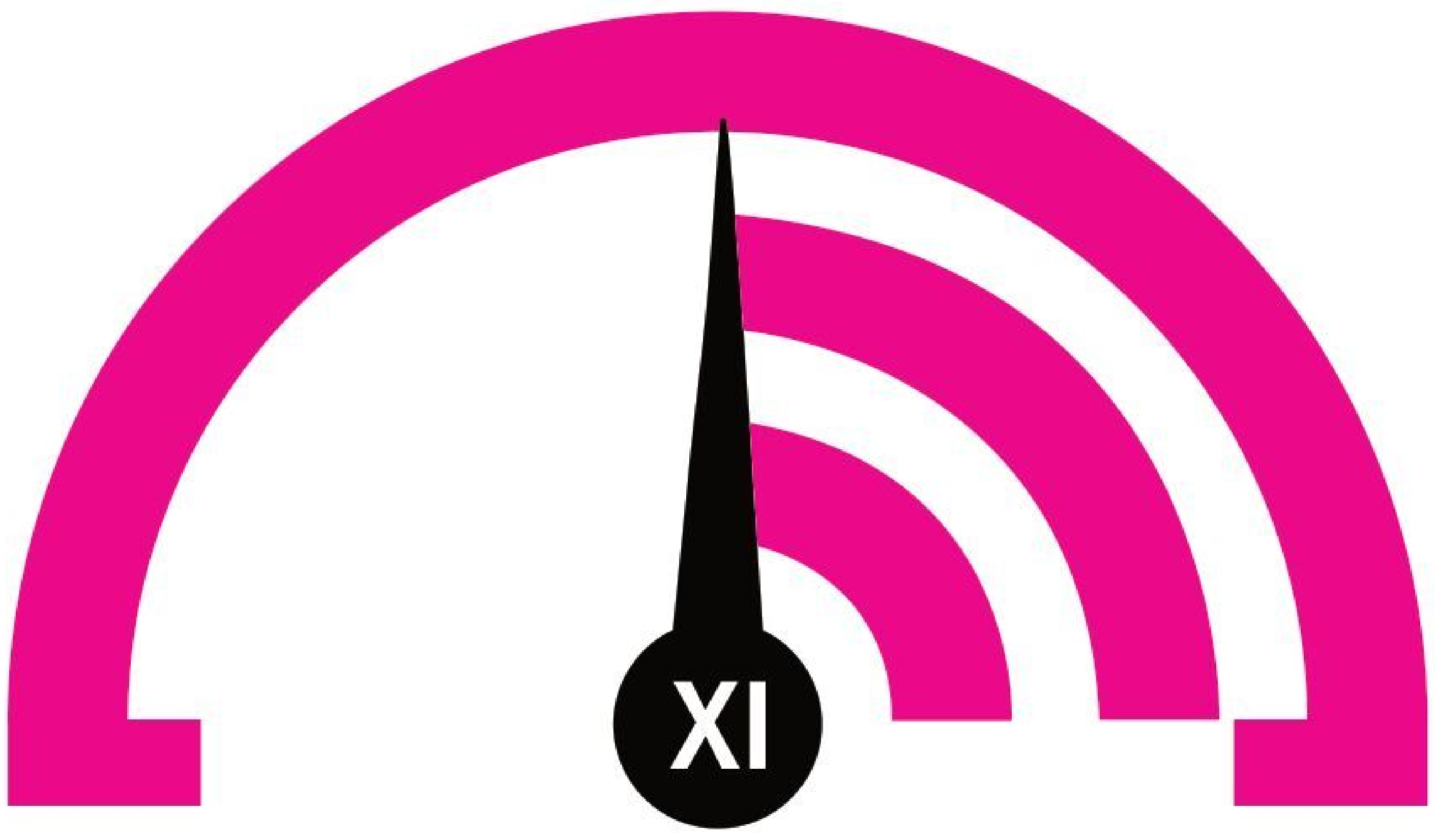
$\therefore$ Required number of days *i.e.*, $16=\frac{n(30-10)}{n+50}$

$\Rightarrow (n + 50) \times 16 = 20n \Rightarrow n = 200$

29. (a) **30. (c)**

◆

MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 1 : Sets, Relations and Functions

Time Taken : 60 Min.

Only One Option Correct Type

1. The range of the function $f(x)=\dfrac{x^2-3x+2}{x^2+x-6}$ is

(a) $R-\{1\}$ (b) $R-\left\{1,\dfrac{1}{5}\right\}$

(c) R (d) $R-\left\{\dfrac{1}{5}\right\}$.

2. The domain of definition of $f(x)=\dfrac{\log_2(x+3)}{x^2+3x+2}$ is

(a) $R-\{-1,-2\}$ (b) $(-2,\infty)$

(c) $R-\{-1,-2,-3\}$ (d) $(-3,\infty)-\{-1,-2\}$.

3. If $A=\left\{(x,y): y=\dfrac{4}{x}, 0\neq x\in R\right\}$

and $B=\{(x,y): y=x, x>0, x\in R\}$

(a) $A\cap B=\phi$ (b) $A\cap B$ is singleton set

(c) $A\cap B$ has two elements

(d) none of these

4. If $f(x)$ is an even function in R and

$$f(x)=\begin{cases}-x, & 0\le x\le 1\\ 1, & 1<x<\infty\end{cases}$$

Then the definition of $f(x)$ in $(-\infty, 0]$ is

(a) $f(x)=\begin{cases}x, & -1\le x\le 0\\ 1, & -\infty<x<-1\end{cases}$

(b) $f(x)=\begin{cases}x, & -1\le x\le 0\\ -1, & -\infty<x<-1\end{cases}$

(c) $f(x)=\begin{cases}-x, & -1\le x\le 0\\ -1, & -\infty<x<-1\end{cases}$

(d) none of these

5. The range of the function $f(x)=\dfrac{1}{2-\cos 3x}$ is

(a) $\left[-\dfrac{1}{3},0\right]$ (b) R

(c) $\left[\dfrac{1}{3},1\right]$ (d) none of these

6. If S in the set of all real x such that $\dfrac{2x-1}{x^3+2x^2+x}>0$, then S contains which of the following open intervals

(a) $\left(-\dfrac{3}{2},\dfrac{1}{2}\right)$ (b) $\left(-\dfrac{3}{2},\dfrac{1}{4}\right)$

(c) $\left(-\dfrac{1}{4},\dfrac{1}{2}\right)$ (d) $\left(\dfrac{1}{2},3\right)$.

One or More Than One Option(s) Correct Type

7. The domain of the function $f(x)=\sqrt{\dfrac{x^2-1}{x-2}}$ is

(a) $(2,\infty)$

(b) $[1,\infty)$

(c) $[-1,1]\cup(2,\infty)$

(d) none of these

8. Let $f(x)=1+\sqrt{x}$ and $g(x)=\dfrac{2x}{x^2+1}$ then

(a) dom $(f+g)=(-1,\infty)$

(b) dom $(f+g)=[0,\infty)$

(c) range of $f\cap$ range of $g=\{1\}$

(d) range of $f\cup$ range of $g=[-1,\infty)$

9. Let $g(x)$ be a function defined on $[-1, 1]$. If the area of the equilateral triangle with two of its vertices at $(0, 0)$ and $(x, g(x))$ is $\sqrt{3}/4$, then the function $g(x) =$

(a) $\pm\sqrt{1+x^2}$ (b) $\sqrt{1-x^2}$

(c) $-\sqrt{1-x^2}$ (d) $\sqrt{1+x^2}$

10. If $e^x + e^{f(x)} = e$, then for $f(x)$

(a) domain = $(-\infty, 1)$ (b) range = $(-\infty, 1)$

(c) domain = $(-\infty, -1)$ (d) range = $(-\infty, -1)$

11. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following are the subsets of A?

(a) $\{3, 4\}$ (b) $\{2, 5\}$

(c) $\{1, 3, 4\}$ (d) $\{\{3, 4\}\}$

12. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is

(a) 211 (b) 256 (c) 220 (d) 219

13. If $f(x) = \sqrt{x^2 - |x|}$, $g(x) = \dfrac{1}{\sqrt{9-x^2}}$, then D_{f+g} contains

(a) $(-3, -1)$ (b) $[1, 3)$

(c) $[-3, 3]$ (d) $\{0\} \cup [1, 3)$.

Comprehension Type

- Generally real functions in calculus are described by some formula and their domains are not explicitly stated. In such cases to find the domain of a function. We use the fact that the domain is the set of all real numbers x for which $f(x)$ is a real number.
- The range of a function $f(x)$ is the set of values of $f(x)$ which it attains at points in its domain. For a real function the co-domain is always a subset of R. So, range of a real function f is the set of all points y such that $y = f(x)$, where $x \in \text{dom } f(x)$.

14. Find the domain of definition of the function $f(x) = \log_4(\log_5(\log_3(18x - x^2 - 77)))$

(a) $(8, 10)$ (b) $(10, 8)$

(c) $(18, 77)$ (d) $(77, 18)$.

15. Find the range of the function

$$f(x) = \sin\left\{\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right\}$$

(a) $[-1, 0]$ (b) $[1, 2]$ (c) $[-1, 1]$ (d) $[1, 5]$.

Matrix Match Type

16. Consider the set $A = \{1, 2, 3, 4, 5, \ldots, n\}$. Then

	Column I		Column II
(P)	Number of subsets of A is	(1)	n^n
(Q)	Number of functions that can be defined from A to A is	(2)	2^n
(R)	Number of relations that can be defined on A is	(3)	2^{n^2}
		(4)	$n!$

	P	Q	R
(a)	2	3	1
(b)	2	1	3
(c)	1	2	4
(d)	1	4	2

Integer Answer Type

17. If $f(x) = \cos(\log x)$, then $f(x)f(y) - \dfrac{1}{2}\left[f\left(\dfrac{x}{y}\right) + f(xy)\right]$ has the value _____.

18. If the range of $y = \dfrac{1}{2 + \sin 3x + \cos 3x}$ is $\left[\dfrac{1}{n+\sqrt{n}}, \dfrac{1}{n-\sqrt{n}}\right]$, then the value of n is _____.

19. Domain of $f(x) = \sqrt{\log(2x - x^2)}$ is $\{p\}$, then p is equal to _____.

20. Suppose that $A_1, A_2, A_3, \ldots\ldots., A_{30}$ are thirty sets, each containing 6 elements and $B_1, B_2, B_3, \ldots\ldots, B_n$ are n sets, each containing 3 elements. If $\bigcup_{i=1}^{30} A_i = S = \bigcup_{i=1}^{n} B_i$ and each element of S belongs to exactly 10 A_i's and to exactly 9 B_i's then $n = 9p$, where $p =$ _____.

◆ ◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……

No. of questions correct ……

Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

CLASS-XI

Chapterwise practice questions for CBSE Exams as per the pattern and marking scheme issued by CBSE for the academic session 2021-22.

Series 1 Sets

Time Allowed : 3 hours **Maximum Marks : 80**

GENERAL INSTRUCTIONS

(1) This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
(2) Part-A has Objective Type Questions and Part -B has Descriptive Type Questions.
(3) Both Part A and Part B have internal choices.

Part-A :

(1) It consists of two sections- I and II.
(2) Section I comprises of 16 very short answer type questions.
(3) Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part-B :

(1) It consists of three sections- III, IV and V.
(2) Section III comprises of 10 questions of 2 marks each.
(3) Section IV comprises of 7 questions of 3 marks each.
(4) Section V comprises of 3 questions of 5 marks each.
(5) Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART-A

SECTION - I

1. Write the roster form of the set A = {the set of all letters of the word TRIGONOMETRY.

OR

Write the following set in set builder form: $A = \{5, 8, 11, 14, 17, 20, 23\}$.

2. Write the following in set builder form: $\{1, 4, 9, ..., 100\}$.

3. Is the collection of ten most talented writers of India is a set or not?

OR

Write the set $\{3, 9, 27, 81\}$ in set builder form.

4. Write all subsets of set $A = \{1, 2, 3\}$.

5. Find the number of subsets of the set $A = \{1, 4, 5\}$.

OR

Given $A = \{x : x$ is a letter of the word MATHEMATICS$\}$. Write down the power set of the set of vowels in A.

6. Find the power sets of the following sets :
$A = \{\{\phi\}\}$, $B = \{1, 2, \{3, 4\}\}$

7. Find $A \cap (B \cup C)$ if $A = \{1, 3, 5, 8\}$, $B = \{3, 5, 7\}$ and $C = \{2, 4, 6, 8\}$

8. Let $U = \{x : x \le 10, x \in N\}$, $A = \{x : x \in N, x \text{ is prime}\}$, $B = \{x : x \in N, x \text{ is even}\}$, then write $A \cap B'$ in roster form.

OR

If $A = \{a, \{b\}\}$, then find $P(A)$.

9. If $U = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $A = \{2, 4, 7\}$, $B = \{3, 5, 7, 9, 11\}$ and $C = \{7, 8, 9, 10, 11\}$, then compute $(A \cap U) \cap (B \cup C)$.

10. If $A = \{1, 2, 3, 4\}$; $B = \{3, 4, 5\}$; $C = \{4, 5, 7, 8\}$; then find $A - (B - C)$.

11. If $A = \{x : x = 4n + 1, 2 \le n \le 5, n \in N\}$, find the then number of subsets of A.

12. If Y is the smallest set such that
$Y \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$, then find the set Y.

OR

If A and B are two sets, then find the value of
$(A - B) \cup (B - A) \cup (A \cap B)$.

13. List all the elements of the set $A = \{x : x$ is a letter of the word HARYANA$\}$.

14. Write the following set in the roster form :
$\{x : x^2 + 7x - 8 = 0, x \in R\}$.

15. Determine $P(A)$ for set $A = \{2, \{1, 0\}\}$.

16. Write the set-builder form of $\{2, 5\}$.

SECTION - II

Both the case study based questions are compulsory. Attempt any 4 sub-parts from each question 17 and 18. Each sub-part carries 1 mark.

17. Rohini's mother organised a small party for her birthday and invite 25 persons. She decided to distribute three types of drinks, Milk,Coffee and Tea. It was found that in party 15 persons had taken Milk, 12 had taken Coffee and 11 had taken Tea, 5 persons had taken both Milk and Tea, 9 had taken both Milk and Coffee, 3 had taken both Coffee and Tea and 3 persons had taken all the three drinks.

Based on the above information, answer the following questions.

(i) The number of persons, who had taken only milk is
(a) 4 (b) 2
(c) 6 (d) 3

(ii) The number of persons, who had taken only coffee is
(a) 2 (b) 6
(c) 3 (d) 4

(iii) The number of persons, who had taken only Tea is
(a) 7 (b) 6
(c) 8 (d) 10

(iv) The number of persons, who had taken at least one of the three drinks, is
(a) 19 (b) 18
(c) 16 (d) 24

(v) The number of persons, who had not taken any of the three drinks, is
(a) 7 (b) 1
(c) 2 (d) 3

18. A survey was conducted in a classroom of 50 students of Class XI on the use of social networking sites. It was found that 15 students are using Facebook, 16 students are using Twitter, 16 students are using Instagram. 8 students are using both Facebook and Twitter, 10 students are using both Twitter and Instagram, 7 students are using both Facebook and Instagram. 5 students are using all the three Social Networking sites.

Based on the above information, answer the following questions.

(i) How many students are using Twitter only?
(a) 3 (b) 4
(c) 5 (d) 2

(ii) How many students are using Facebook and Twitter but not Instagram?

(a) 4 (b) 3
(c) 2 (d) 5

(iii) How many students are using Twitter and Instagram but not Facebook?

(a) 4 (b) 3
(c) 2 (d) 5

(iv) How many students are using at least one of the three social networking sites?

(a) 25 (b) 27
(c) 50 (d) 49

(v) How many students are not using any of the three social networking sites?

(a) 21 (b) 25
(c) 23 (d) 27

PART-B

SECTION-III

19. Let $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6\}$. Then find the number of sets C such that $A \cap B \subseteq C \subseteq A \cup B$.

20. If $A = \{1\}$ and $B = \{2, 3\}$ are two non-empty sets, then find the value of $P(A) \cup P(B)$.

OR

Let X and Y be the sets of all positive divisors of 400 and 1000 respectively (including 1 and the number), then find $n(X \cap Y)$.

21. Show that set $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to $B \cap C'$.

22. If $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$ and $A = \{x : x^2 - 5x + 6 = 0\}$ and $B = \{x : x^2 - 3x + 2 = 0\}$, then find $(A \cap B)'$.

OR

Express the given set $M = \{x : -4 < x \leq 7, x \in R\}$ in interval form and represent it on the number line.

23. Is the following pair of sets equal? Justify your answer.

$A = \{x : x \in Z \text{ and } x^2 \leq 4\}$,
$B = \{x : x \in R, x^2 - 3x + 2 = 0\}$.

24. If $A = \{2x : x \in N \text{ and } 1 \leq x < 4\}$,
$B = \{(x + 2) : x \in N \text{ and } 2 \leq x < 5\}$ and
$C = \{x : x \in N \text{ and } 4 < x < 8\}$, find
(i) $A \cup B$
(ii) $(A \cup B) \cap C$

25. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$, find
(i) $B \cap (C \cup D)$ (ii) $A \cap (B \cup C)$

OR

Draw the Venn diagram of $(A \cup C)' - B$.

26. If $A \cap B' = \phi$, then show that $A = A \cap B$, and hence show that $A \subseteq B$.

27. Using properties of sets, show that $(A \cap B) \cup (A - B) = A$.

28. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$, then find $(A - B)'$.

SECTION-IV

29. From the sets given below, select the equivalent sets :

$A = \{1, 2, 3, 5\}$, $B = \{5, 10\}$, $C = \{x : x^2 - 5x + 4 = 0\}$,
$D = \{x : x \text{ is a letter of the word LOVE}\}$
$E = \{x : x \text{ is a letter of the word WOLF}\}$.

30. If $A = \{(x, y) : x^2 + y^2 = 25\}$ and $B = \{(x, y) : x^2 + 16y^2 = 144\}$, then find the number of points that $A \cap B$ contains.

31. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$, then find $(A \cup B)'$, $(A' \cap B')$ and $(A \Delta B)$.

OR

Let $A = \{2, 3, 4, 5\}$, $B = \{2, 3, 4\}$ and $C = \{2, 3, 5\}$. Find all the sets X satisfying
(i) $X \subset A, X \not\subset B$
(ii) $X \subset B, X \neq B, X \not\subset C$

32. If $A = \{x : x = 4^n - 3n - 1 \text{ and } n \in N\}$
$B = \{y : y = 9(n - 1) \text{ and } n \in N\}$.
Prove that $A \subset B$.

33. If $A = \{1, 3, 5, 6, 7\}$, $B = \{2, 3, 6, 8\}$ and $C = \{1, 2, 3, 4\}$, then find
(i) $A \cup (B \cap C)$
(ii) $A - (B \cup C)$
(iii) $(A \cap C) \cup C$

34. Let A and B be two sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that $A = B$.

OR

For any three sets A, B and C, prove the following (by using different laws on operations of sets) :
(i) $(A - B) \cap (B - A) = \phi$
(ii) $A - B = B' - A'$.

35. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$, $B = \{2, 3, 5, 8\}$. Verify that

(i) $(A \cup B)^C = A^C \cap B^C$

(ii) $(A \cap B)^C = A^C \cup B^C$.

SECTION - V

36. Are the following equal?

(i) $A = \{2, 1\}$, $B = \{1, 2\}$,
$C = \{x : x^2 - 3x + 2 = 0\}$.

(ii) $A = \{x : x^3 - 8 = 0$ and x is a real number$\}$
$B = \{x : x^2 + 7x - 18 = 0$ and $x > 0\}$

(iii) A = The set of letters in the word 'ALLOY'.
B = The set of letters in the word 'LOYAL'.

(iv) $A = \{x : x \in Z$ and $x^2 < 4\}$
$B = \{x : x \in R$ and $x^2 - 3x + 2 = 0\}$.

(v) $A = \{x : x$ is a multiple of $10\}$
$B = \{10, 15, 20, 25, 30, ...\}$

OR

Draw the Venn diagrams to illustrate the following relationship among sets, E, M and U, where E is the set of students studying English in a school, M is the set of students studying Mathematics in the same school, U is the set of all students in that school.

(i) All the students who study Mathematics study English, but some students who study English do not study Mathematics.

(ii) There is no student who studies both Mathematics and English.

(iii) Some of the students study Mathematics but do not study English, some study English but do not study Mathematics, and some study both.

(iv) Not all students study Mathematics, but every student studying English studies Mathematics.

37. In a class, 18 students offered physics, 23 offered chemistry and 24 offered mathematics. Of these, 13 are in both chemistry and mathematics, 12 in physics and chemistry, 11 in mathematics and physics, and 6 in all the three subjects. Find

(i) how many students are there in the class?

(iii) how many students are taking exactly one of the three subjects?

OR

Show that:

(i) $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

(ii) $(A \cup B) - (A - B) = B$.

38. For any sets A, B and C using properties of sets, prove that

(i) $A - (A - B) = A \cap B$

(ii) $(A - B) \cap (A - C) = A - (B \cup C)$

OR

A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all three sports. How many medals received in exactly two of three sports?

SOLUTIONS

1. Roster form of A is
$A = \{T, R, I, G, O, N, M, E, Y\}$

OR

We have $A = \{5, 8, 11, 14, 17, 20, 23\}$
Set builder form of A is $A = \{x : x = 3n + 2, n \in N$ and $1 \le n \le 7\}$

2. Let $A = \{1, 4, 9, ..., 100\}$
All elements of the set are perfect squares.
$\therefore\ A = \{x : x = n^2, n \in N$ and $1 \le n \le 10\}$

3. The collection of most talented writers of India is not well defined because opinions about 'most talented writers' vary from person to person and hence it does not form a set.

OR

Let $A = \{3, 9, 27, 81\}$
Thus, $A = \{x : x = 3^n, n \in N$ and $1 \le n \le 4\}$

4. Subsets of set A are ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, $\{1, 2, 3\}$

5. The number of subsets of set $A = 2^3 = 8$.

OR

$A = \{M, A, T, H, E, I, C, S\}$
Set of vowels in $A = \{A, E, I\}$
Thus, power set of vowels in $A = \{\phi, \{A\}, \{E\}, \{I\}, \{A, E\}, \{E, I\}, \{A, I\}, \{A, E, I\}\}$.

6. $P(A) = \{X : X \subseteq A\} = \{X : X = \phi, X = A\}$
$= \{\phi, A\} = \{\phi, \{\{\phi\}\}\}$
$P(B) = \{X : X \subseteq B\}$
$= \{\phi, \{1\}, \{2\}, \{\{3, 4\}\}, \{1, 2\}, \{1, \{3, 4\}\}, \{2, \{3, 4\}\}, B\}$

7. $A \cap (B \cup C) = \{1, 3, 5, 8\} \cap [\{3, 5, 7\} \cup \{2, 4, 6, 8\}]$
$= \{1, 3, 5, 8\} \cap \{2, 3, 4, 5, 6, 7, 8\}$
$= \{3, 5, 8\}$

8. $U = \{1, 2, 3, ..., 10\}, A = \{2, 3, 5, 7\}$
$B = \{2, 4, 6, 8, 10\}, B' = \{1, 3, 5, 7, 9\}$
$\Rightarrow \quad A \cap B' = \{2, 3, 5, 7\} \cap \{1, 3, 5, 7, 9\} = \{3, 5, 7\}$

OR

Let $B = \{b\}$. Then, $A = \{a, B\}$.
$\therefore \quad P(A) = \{\phi, \{a\}, \{B\}, \{a, B\}\} = \{\phi, \{a\}, \{\{b\}\}, \{a, \{b\}\}\}$.

9. Here $A \cap U = \{2, 4, 7\}, B \cup C = \{3, 5, 7, 8, 9, 10, 11\}$.
$(A \cap U) \cap (B \cup C) = \{2, 4, 7\} \cap \{3, 5, 7, 8, 9, 10, 11\} = \{7\}$

10. $B - C = \{3, 4, 5\} - \{4, 5, 7, 8\} = \{3\}$
$A - (B - C) = \{1, 2, 3, 4\} - \{3\} = \{1, 2, 4\}$

11. Given that, $A = \{x : x = 4n + 1, 2 \le n \le 5, n \in N\}$
$= \{9, 13, 17, 21\}$
$\therefore$ Number of elements in A is 4.
So, number of subsets $= 2^4 = 16$

12. Since $Y \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$, then the set Y must contain 3, 5, 9. Since the set Y is smallest, we have $Y = \{3, 5, 9\}$.

OR

$(A - B) \cup (B - A) \cup (A \cap B)$
= only $A \cup$ only $B \cup$ Both A and $B = A \cup B$

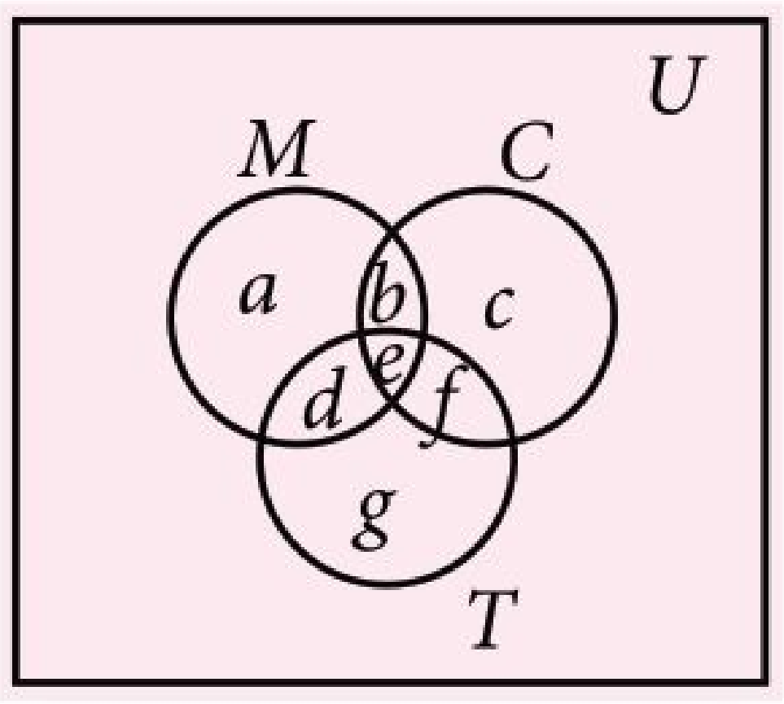
13. $A = \{\text{H, A, R, Y, N}\}$

14. $x^2 + 7x - 8 = 0 \Rightarrow (x + 8)(x - 1) = 0$
$\Rightarrow x = -8$ or $x = 1$
Thus, roster form of given set is $\{-8, 1\}$.

15. $P(A) = \{\phi, \{2\}, \{\{1, 0\}\}, \{2, \{1, 0\}\}\}$

16. The set builder form of (2, 5) is
$\{x : x$ is a prime natural number and divisor of $10\}$

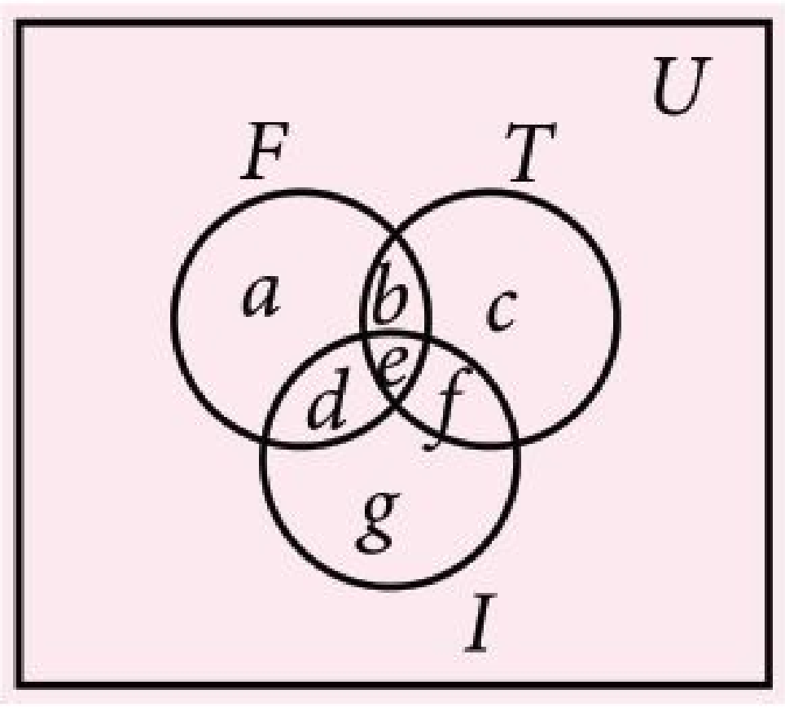
17. Let M, C and T denotes the set of persons, who had taken Milk, Coffee and Tea respectively.



Here, $n(U) = 25, n(M) = 15, n(C) = 12, n(T) = 11$,
$n(M \cap T) = 5, n(M \cap C) = 9, n(C \cap T) = 3$
and $n(M \cap C \cap T) = 3$
Now, $e = 3, b + e = 9 \Rightarrow b = 9 - 3 = 6$,
$d + e = 5 \Rightarrow d = 5 - 3 = 2, e + f = 3 \Rightarrow f = 3 - 3 = 0$
Also, $a + b + d + e = 15 \Rightarrow a = 15 - 6 - 2 - 3 = 4$,
$b + c + e + f = 12 \Rightarrow c = 12 - 6 - 3 - 0 = 3$,
$d + e + f + g = 11 \Rightarrow g = 11 - 2 - 3 - 0 = 6$

(i) (a) : number of persons, who had taken only milk $= a = 4$
(ii) (c) : Number of persons, who had taken only coffee $= c = 3$
(iii) (b) : Number of persons, who had taken only Tea $= g = 6$
(iv) (d) : Number of persons, who had taken atleast one of the three drinks $= n(M \cup T \cup C)$
$= a + b + c + d + e + f + g$
$= 4 + 6 + 3 + 2 + 3 + 0 + 6$
$= 24$
(v) (b) : Number of persons, who had not taken any of the drink $= n(U) - n(M \cup C \cup T)$
$= 25 - 24 = 1$

18. Let F, T and I respectively denotes the set of students using Facebook, Twitter and Instagram.



Here, $n(U) = 50, n(F) = 15, n(T) = 16, n(I) = 16$
$n(F \cap T) = 8, n(T \cap I) = 10$,
$n(F \cap I) = 7, n(F \cap T \cap I) = 5$
Also, $e = 5, b + e = 8 \Rightarrow b = 8 - 5 = 3, d + e = 7 \Rightarrow d = 7 - 5 = 2$,
$e + f = 10 \Rightarrow f = 10 - 5 = 5$
Now, $a + b + e + d = 15 \Rightarrow a = 15 - 3 - 5 - 2 = 5$,
$b + c + f + e = 16 \Rightarrow c = 16 - 3 - 5 - 5 = 3$,
$e + f + g + d = 16 \Rightarrow g = 16 - 5 - 5 - 2 = 4$
(i) (a) : Number of students using Twitter only $= c = 3$
(ii) (c) : Number of students using Facebook and Twitter but not Instagram $= b = 3$
(iii) (d) : Number of students using Twitter and Instagram but not Facebook $= f = 5$
(iv) (b) : Number of students using at least one of the three social networking sites $= n(F \cup T \cup I)$
$= a + b + c + d + e + f + g$
$= 5 + 3 + 3 + 2 + 5 + 5 + 4 = 27$
(v) (c) : Number of students not using any of the social networking sites $= 50 - 27 = 23$

19. $A \cap B \subseteq C \subseteq A \cup B$
$\Rightarrow \quad \{2, 4\} \subseteq C \subseteq \{1, 2, 3, 4, 6\}$
Required number of sets is 8
i.e., $\{2, 4\}, \{1, 2, 4\}, \{2, 4, 3\}, \{2, 4, 6\}, \{1, 2, 4, 3\}$,
$\{3, 2, 4, 6\}, \{2, 4, 1, 6\}, \{1, 2, 3, 4, 6\}$.

20. We have, $A = \{1\}, B = \{2, 3\}$
Now, $P(A) = \{\phi, \{1\}\}, P(B) = \{\phi, \{2\}, \{3\}, \{2, 3\}\}$
$\therefore \quad P(A) \cup P(B) = \{\phi, \{1\}, \{2\}, \{3\}, \{2, 3\}\}$

OR

X and Y be set of all divisors of 400 and 1000 respectively.

So $X = \{1, 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 200, 400\}$

$Y = \{1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 125, 200, ..., 1000\}$

$X \cap Y = \{1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200\}$

12 elements are common in both the sets.

$\Rightarrow \quad n(X \cap Y) = 12$

21. Consider, $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$

$= (A \cup B \cup C) \cap (A' \cup B \cup C) \cap C'$

$= [(A \cap A') \cup (B \cup C)] \cap C'$

$= [(\phi \cup B \cup C)] \cap C' = (B \cup C) \cap C'$

$= (B \cap C') \cup (C \cap C') = (B \cap C') \cup \phi = B \cap C'$

22. $\because \quad U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$

$= \{0, 1, 2, 3\}$

$A = \{x : x^2 - 5x + 6 = 0\} = \{2, 3\}$

and $B = \{x : x^2 - 3x + 2 = 0\} = \{1, 2\}$

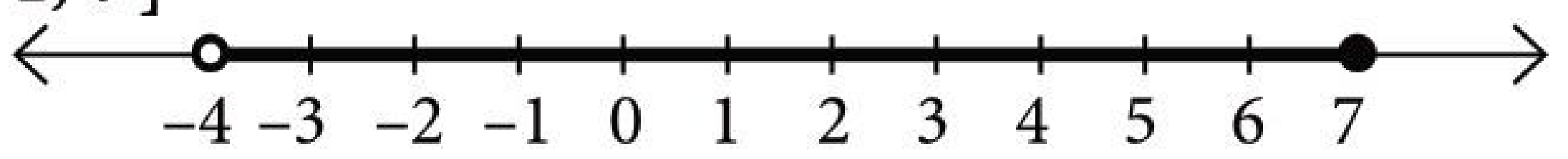
$\therefore \quad A \cap B = \{2\}$

Hence, $(A \cap B)' = U - (A \cap B) = \{0, 1, 2, 3\} - \{2\} = \{0, 1, 3\}$

OR

$M = \{x : -4 < x \le 7, x \in R\}$

$M = (-4, 7]$



23. $A = \{x : x \in Z \text{ and } x^2 \le 4\} = \{-2, -1, 0, 1, 2\}$

$[\because x^2 \le 4 \Rightarrow x \in [-2, 2]]$

$B = \{x : x^2 - 3x + 2 = 0\} = \{1, 2\}$

Since, $0 \in A$ and $0 \notin B$ $\therefore$ A and B are not equal sets.

24. We have $A = \{2,4,6\}$, $B = \{4,5,6\}$ and $C = \{5,6,7\}$

(i) $A \cup B = \{2, 4, 5, 6\}$

(ii) $(A \cup B) \cap C = \{2, 4, 5, 6\} \cap \{5, 6, 7\} = \{5, 6\}$

25. **(i)** $B \cap (C \cup D) = \{7, 9, 11, 13\} \cap \{11, 13, 15, 17\}$

$= \{11, 13\}$

(ii) $A \cap (B \cup C) = \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13, 15\}$

$= \{7, 9, 11\}$

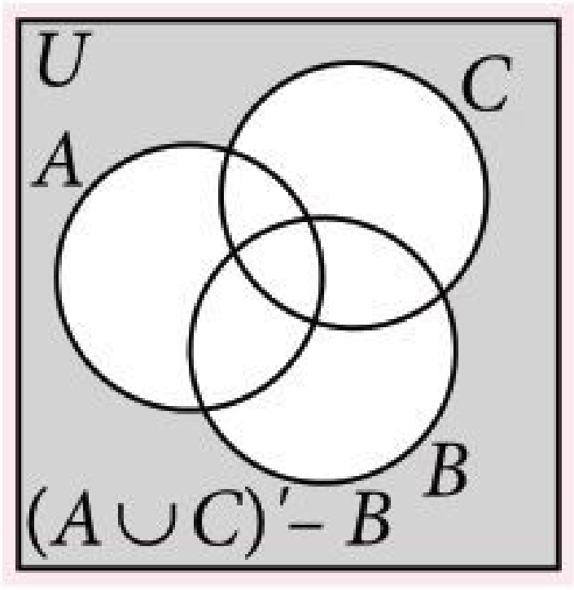
OR

Consider, $(A \cup C)' - B$

$= (A \cup C)' \cap B'$

$= (A \cup C \cup B)'$

$= (A \cup B \cup C)'$



26. If $A \cap B' = \phi$ be given. Then,

$A = A \cap U$, where U is the universal set.

$= A \cap (B \cup B')$ $\quad [\because B \cup B' = U]$

$= (A \cap B) \cup (A \cap B')$ $\quad$ [By Distributive law]

$= (A \cap B) \cup \phi$ $\quad [\because A \cap B' = \phi]$

$= (A \cap B)$

But, $A \cap B = A$ means $A \subseteq B$.

27. Consider, $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$

$= A \cap (B \cup B') = A \cap U = A$

28. $A - B = \{1, 2, 3, 4\} - \{2, 4, 6, 8\} = \{1, 3\}$

$(A - B)' = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 3\}$

$= \{2, 4, 5, 6, 7, 8, 9\}$

29. We have, $A = \{1, 2, 3, 5\}$, $B = \{5, 10\}$,

$C = \{x : x^2 - 5x + 4 = 0\} = \{4, 1\}$,

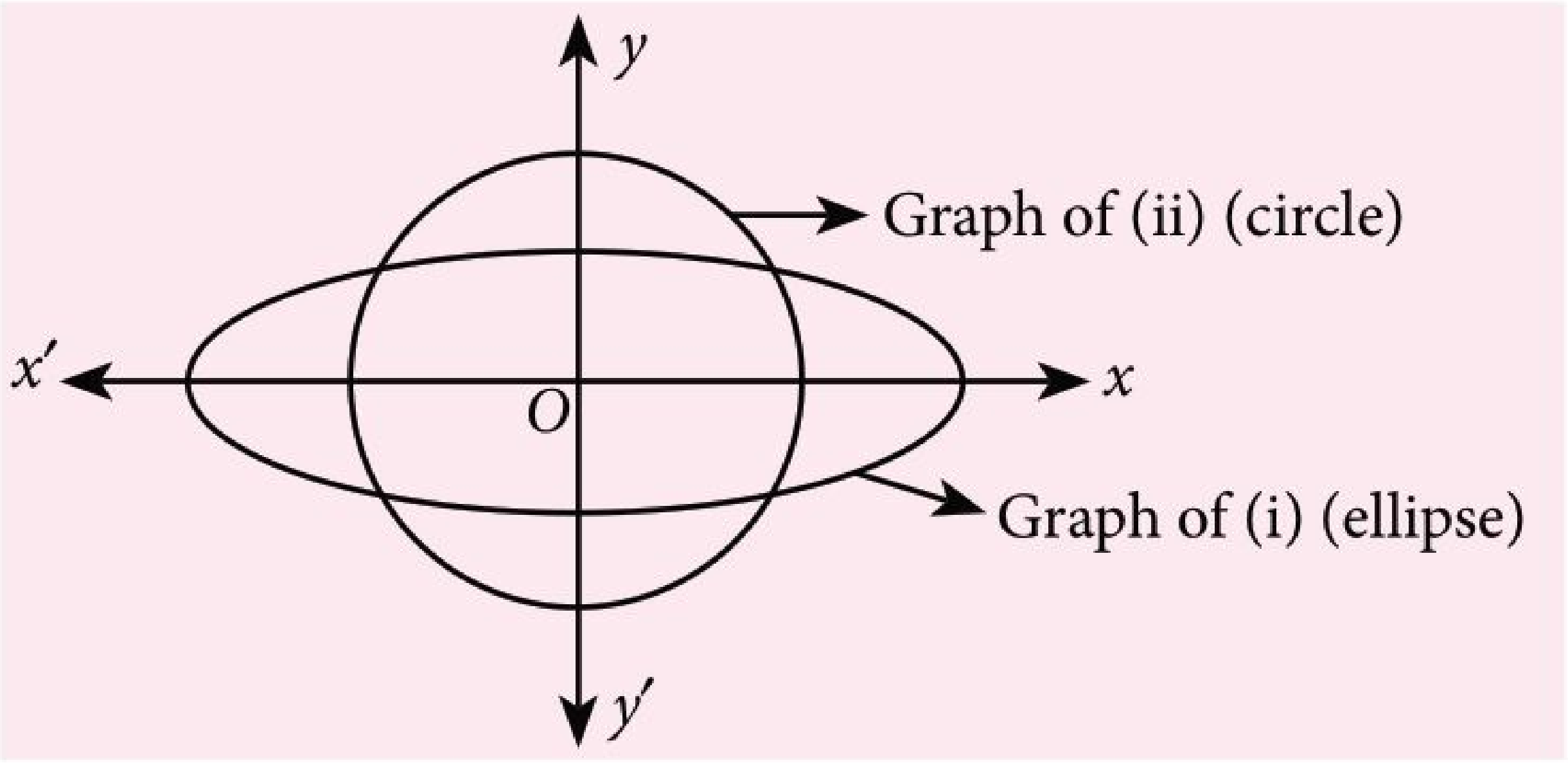
$D = \{L, O, V, E\}$, $E = \{W, O, L, F\}$

Thus, B and C are equivalent sets; Also, A, D, E are equivalent sets.

30. Given $x^2 + 16y^2 = 144$

$\Rightarrow \quad \frac{x^2}{12^2} + \frac{y^2}{3^2} = 1$ (An ellipse whose major axis is x-axis) ...(i)

and $x^2 + y^2 = 25$ (a circle with radius $r = 5$ units) ...(ii)



From the graph of (i) and (ii), we note that $A \cap B$ have four points.

31. Given sets are $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$,

$A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$

Now, $A' = U - A = \{1, 3, 5, 7, 9\}$

and $B' = U - B = \{1, 4, 6, 8, 9\}$

(i) $A \cup B = \{2, 4, 6, 8\} \cup \{2, 3, 5, 7\} = \{2, 3, 4, 5, 6, 7, 8\}$

$\therefore \quad (A \cup B)' = U - (A \cup B) = \{1, 9\}$

(ii) $(A' \cap B') = \{1, 3, 5, 7, 9\} \cap \{1,4, 6, 8, 9\} = \{1, 9\}$

(iii) Now, $A - B = \{2, 4, 6, 8\} - \{2, 3, 5, 7\} = \{4, 6, 8\}$

and $B - A = \{2, 3, 5, 7\} - \{2, 4, 6, 8\} = \{3, 5, 7\}$

$\therefore \quad A \Delta B = (A - B) \cup (B - A) = \{3, 4, 5, 6, 7, 8\}$

OR

(i) Any one of the sets $\{5\}$, $\{2, 5\}$, $\{3, 5\}$, $\{4, 5\}$, $\{2, 3, 5\}$, $\{2, 4, 5\}$, $\{3, 4, 5\}$ and A can be taken as X as each of these is a subset of A but not a subset of B. Note that any subset of A which contains 5 satisfy the desired conditions.

(ii) Any one of the sets $\{4\}$, $\{2, 4\}$, $\{3, 4\}$ are subsets of B which satisfy all the required properties. Hence, any one of these can be taken as X. In this case, any proper subset of B containing 4 satisfying the desired conditions.

32. $x = 4^n - 3n - 1 = (1 + 3)^n - 3n - 1$

$= 1 + {}^nC_1 \cdot 3 + {}^nC_2 \cdot 3^2 + {}^nC_3 \cdot 3^3 + ... + {}^nC_n \cdot 3^n - 3n - 1$

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$= 9\,^nC_2 + 27\,^nC_3 + 81\,^nC_4 + \ldots 3^n\,^nC_n$

$\therefore A = \{9(^nC_2 + 3\cdot{}^nC_3 + 3^2\cdot{}^nC_4 + \ldots + 3^{n-2}) : n \in N\}$

and $B = \{9(n-1) : n \in N\}$ $\therefore y = 9(n-1)$

For $n = 1, x = 4 - 3 - 1 = 0$ and $y = 0$

For $n = 2, x = 9\cdot{}^2C_2 = 9$ and $y = 9(2-1) = 9$

Also, for $n = 3$,

$x = 9(^3C_2 + 3\cdot{}^3C_3) = 54$ and $y = 9(n-1) = 9\cdot 2 = 18$

Thus A contain 0 and some positive integral multiples of 9, while B contains 0 and all positive integral multiples of 9.

Hence, $A \subset B$.

33. **(i)** $B \cap C = \{2, 3\}$

Now, $A \cup (B \cap C) = \{1, 2, 3, 5, 6, 7\}$

(ii) $B \cup C = \{1, 2, 3, 4, 6, 8\}$

Now, $A - (B \cup C) = \{1, 3, 5, 6, 7\} - \{1,2,3,4,6,8\}$

$= \{5, 7\}$

(iii) $A \cap C = \{1, 3\}$

$\therefore (A \cap C) \cup C = \{1, 3\} \cup \{1, 2, 3, 4\} = \{1, 2, 3, 4\}$

34. Here, $A \cup X = B \cup X$ for some set X

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow A = (A \cap B) \cup (A \cap X)$ $[\because A \cap (A \cup X) = A]$

$\Rightarrow A = (A \cap B) \cup \phi \Rightarrow A = A \cap B$ ...(i)

Also, $A \cup X = B \cup X$

$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow (B \cap A) \cup (B \cap X) = B$ $[\because B \cap (B \cup X) = B]$

$\Rightarrow (B \cap A) \cup \phi = B$ $[\because B \cap X = \phi]$

$\Rightarrow B \cap A = B$ ...(ii)

From (i) and (ii), we get, $A = B$.

OR

(i) $(A - B) \cap (B - A) = (A \cap B') \cap (B \cap A')$

$= A \cap (B' \cap B) \cap A' = A \cap \phi \cap A' = A \cap (\phi \cap A')$

$= A \cap \phi = \phi$

(ii) $(A - B) = (A \cap B') = B' \cap A = B' \cap (A')' = B' - A'$

35. $A^C = \{2, 4, 6, 8\}^C = \{1, 3, 5, 7, 9\}$

$B^C = \{2, 3, 5, 8\}^C = \{1, 4, 6, 7, 9\}$

Also, $A \cup B = \{2, 4, 6, 8\} \cup \{2, 3, 5, 8\}$

$= \{2, 3, 4, 5, 6, 8\}$

$A \cap B = \{2, 4, 6, 8\} \cap \{2, 3, 5, 8\} = \{2, 8\}$

(i) L.H.S. $= (A \cup B)^C = \{1, 7, 9\}$

R.H.S. $= A^C \cap B^C = \{1, 7, 9\}$

Hence, $(A \cup B)^C = A^C \cap B^C$

(ii) L.H.S. $= (A \cap B)^C = \{2, 8\}^C = \{1, 3, 4, 5, 6, 7, 9\}$

R.H.S. $= A^C \cup B^C = \{1, 3, 4, 5, 6, 7, 9\}$

Hence, $(A \cap B)^C = A^C \cup B^C$

36. **(i)** We have, $A = \{2, 1\}, B = \{1, 2\}, C = \{1, 2\}$

$[\because x^2 - 3x + 2 = 0 \Rightarrow (x-1)(x-2) = 0]$

$\therefore A = B = C$

(ii) We have, $A = \{2\}$

$\{\because x^3 - 8 = 0 \Rightarrow x^3 = 2^3 \Rightarrow x = 2\}$

$B = \{2\}$ $[\because x^3 + 7x - 18 = 0 \Rightarrow (x-2)(x+9) = 0$

$\Rightarrow x = 2, x = -9$ But $x > 0 \therefore x = 2]$

$\therefore A = B$

(iii) $A = \{A, L, O, Y\}, B = \{L, O, Y, A\}$

$\therefore A = B$

(iv) We have, $A = \{-1, 0, 1\}$

$[\because x^2 < 4 \Rightarrow x \in (-2, 2)]$

$B = \{1, 2\}$ $[\because x^2 - 3x + 2 = 0 \Rightarrow (x-1)(x-2) = 0$

$\Rightarrow x = 1, 2]$

As $0 \in A$ & $0 \notin B$ $\therefore A \neq B$.

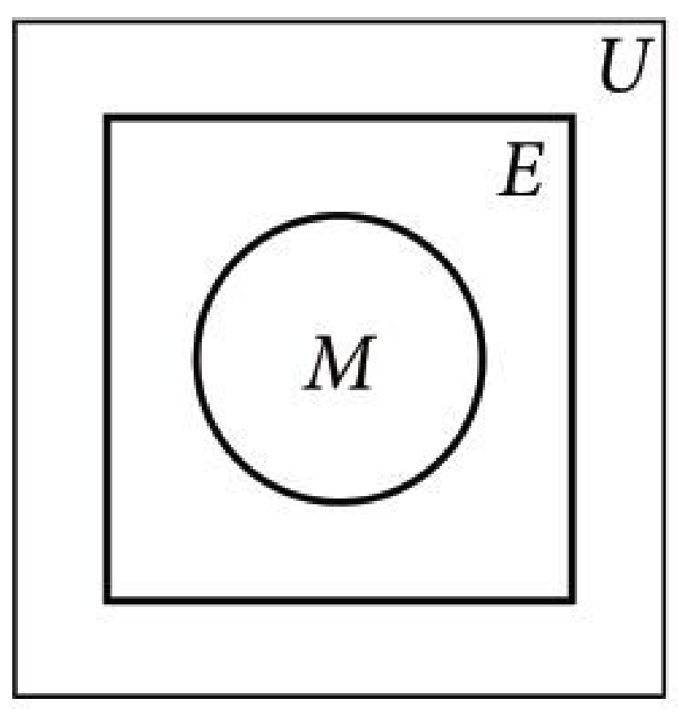
(v) We have, $A = \{10, 20, 30\ldots\}$

$B = \{10, 15, 20, 25, 30 \ldots\}$

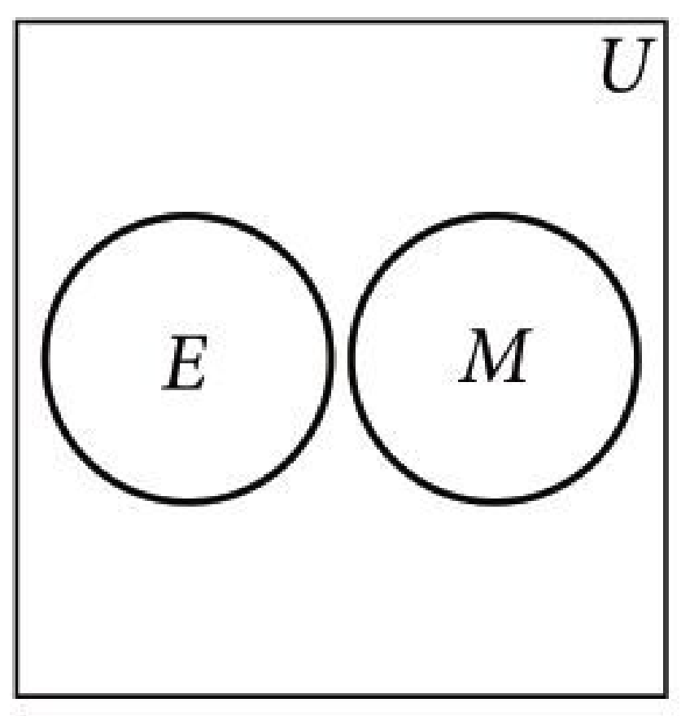
As $15 \in B$ but $15 \notin A$ $\therefore A \neq B$

OR

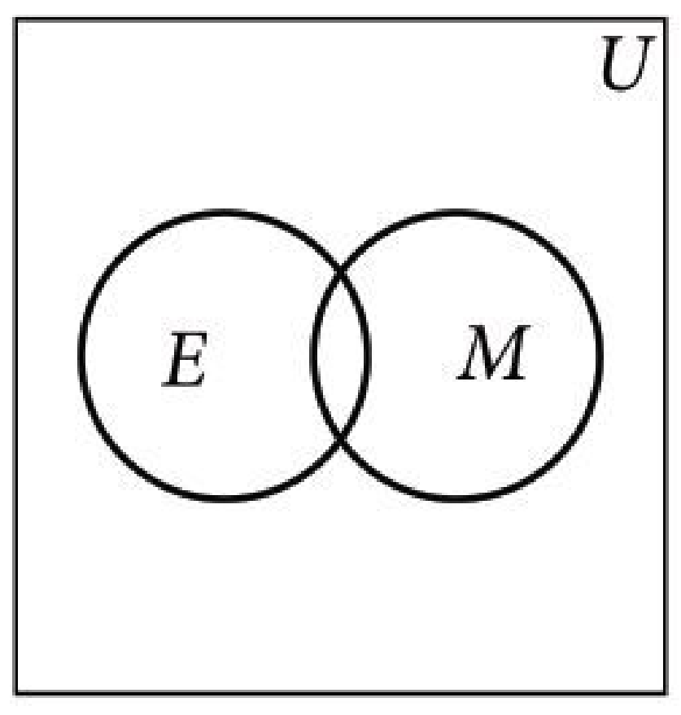
(i)



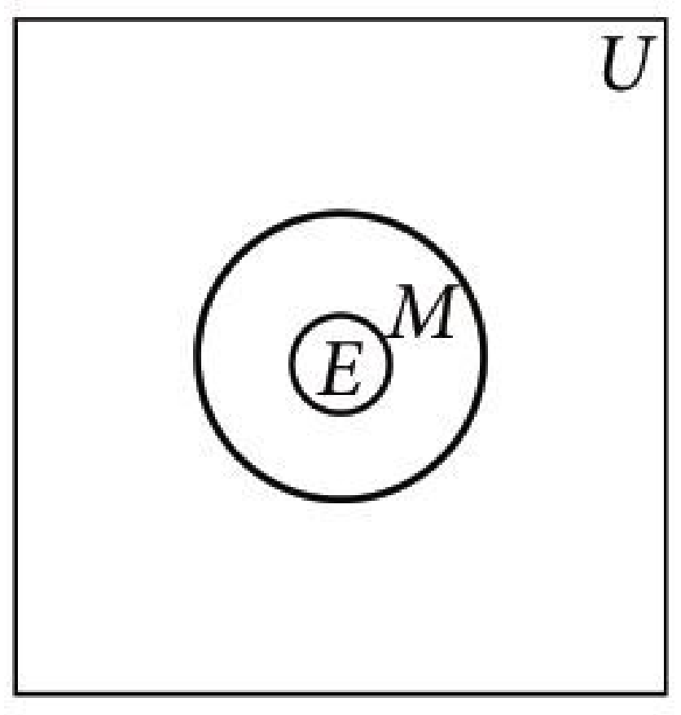
(ii)



(iii)



(iv)



37. Let P, C and M be the set of students who offered Physics, Chemistry and Mathematics respectively.

$n(P) = 18, n(C) = 23, n(M) = 24, n(C \cap M) = 13, n(P \cap C) = 12, n(M \cap P) = 11, n(P \cap C \cap M) = 6$

(i) Number of students in the class

$= n(P \cup C \cup M) = n(P) + n(C) + n(M) - n(P \cap C) - n(C \cap M) - n(M \cap P) + n(P \cap C \cap M)$

$= 18 + 23 + 24 - 12 - 13 - 11 + 6 = 35$

(ii) Number of students who were offered Mathematics but not Chemistry $= n(M \cap C')$

$= n(M) - n(M \cap C) = 24 - 13 = 11$

(iii) Number of students who are taking exactly one of the three subjects

$= n(P \cup C \cup M) - n(P \cap C) - n(C \cap M) - n(M \cap P) + 2n(P \cap C \cap M)$

$= 35 - 12 - 13 - 11 + 2(6) = 11$

OR

(i) $(A \cup B) - (A \cap B) = (A \cup B) \cap (A \cap B)^c$

$(\because A - B = A \cap B^c)$

$= (A \cup B) \cap (A^c \cup B^c)$ (By De-Morgan's law)

$= \{(A \cup B) \cap A^c\} \cup \{(A \cup B) \cap B^c\}$ (By Distributive law)

$= \{(A \cap A^c) \cup (B \cap A^c)\} \cup \{(A \cap B^c) \cup (B \cap B^c)\}$

(By Distributive laws)

$= \{\phi \cup (B \cap A^c)\} \cup \{(A \cap B^c) \cup \phi\}$

$= (B \cap A^c) \cup (A \cap B^c)$ $\qquad (\because\ A \cup \phi = A)$

$= (B - A) \cup (A - B) = (A - B) \cup (B - A)$

(ii) $(A \cup B) - (A - B) = (A \cup B) \cap (A - B)'$

$= (A \cup B) \cap (A \cap B')'$

$= (A \cup B) \cap (A' \cup B)$ $\qquad$ (by De-Morgan's laws)

$= (A \cap A') \cup B = \phi \cup B = B.$

38. **(i)** L.H.S. $= A - (A - B) = A - (A \cap B')$

$[\because A - B = A \cap B']$

$= A \cap (A \cap B')' = A \cap [A' \cup (B')']$

[$\because$ By De-Morgan's law]

$= A \cap (A' \cup B)$ $\qquad [\because (A')' = A]$

$= (A \cap A') \cup (A \cap B)$ $\qquad$ [Using Distributive law]

$= \phi \cup (A \cap B) = A \cap B =$ R.H.S.

(ii) Let $x \in (A - B) \cap (A - C)$ ...(i)

$\Rightarrow x \in (A - B)$ and $x \in (A - C)$

$\Rightarrow (x \in A$ and $x \notin B)$ and $(x \in A$ and $x \notin C)$

$\Rightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Rightarrow x \in A$ and $x \notin (B \cup C)$

$\Rightarrow x \in A - (B \cup C)$ ...(ii)

From (i) and (ii), we get

$(A - B) \cap (A - C) \subset A - (B \cup C)$ ...(iii)

Now, let $y \in A - (B \cup C)$ ...(iv)

$\Rightarrow y \in A$ and $y \notin (B \cup C)$

$\Rightarrow y \in A$ and $(y \notin B$ and $y \notin C)$

$\Rightarrow (y \in A$ and $y \notin B)$ and $(y \in A$ and $y \notin C)$

$\Rightarrow y \in (A - B)$ and $y \in (A - C)$

$\Rightarrow y \in (A - B) \cap (A - C)$ ...(v)

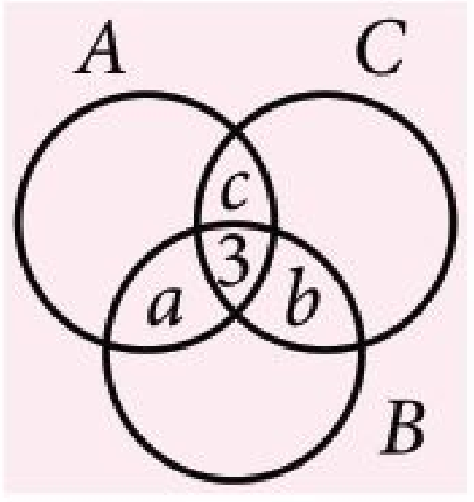
From (iv) and (v)

$A - (B \cup C) \subset (A - B) \cap (A - C)$ ...(vi)

From (iii) and (vi), we get

$A - (B \cup C) = (A - B) \cap (A - C)$

OR



Let A : Football,
B : Basketball, C : Cricket
Let a, b, c represents men getting medals in exactly two sports.

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$\Rightarrow 58 = 38 + 15 + 20 - \{n(A \cap B) + n(B \cap C) + n(C \cap A)\} + 3$

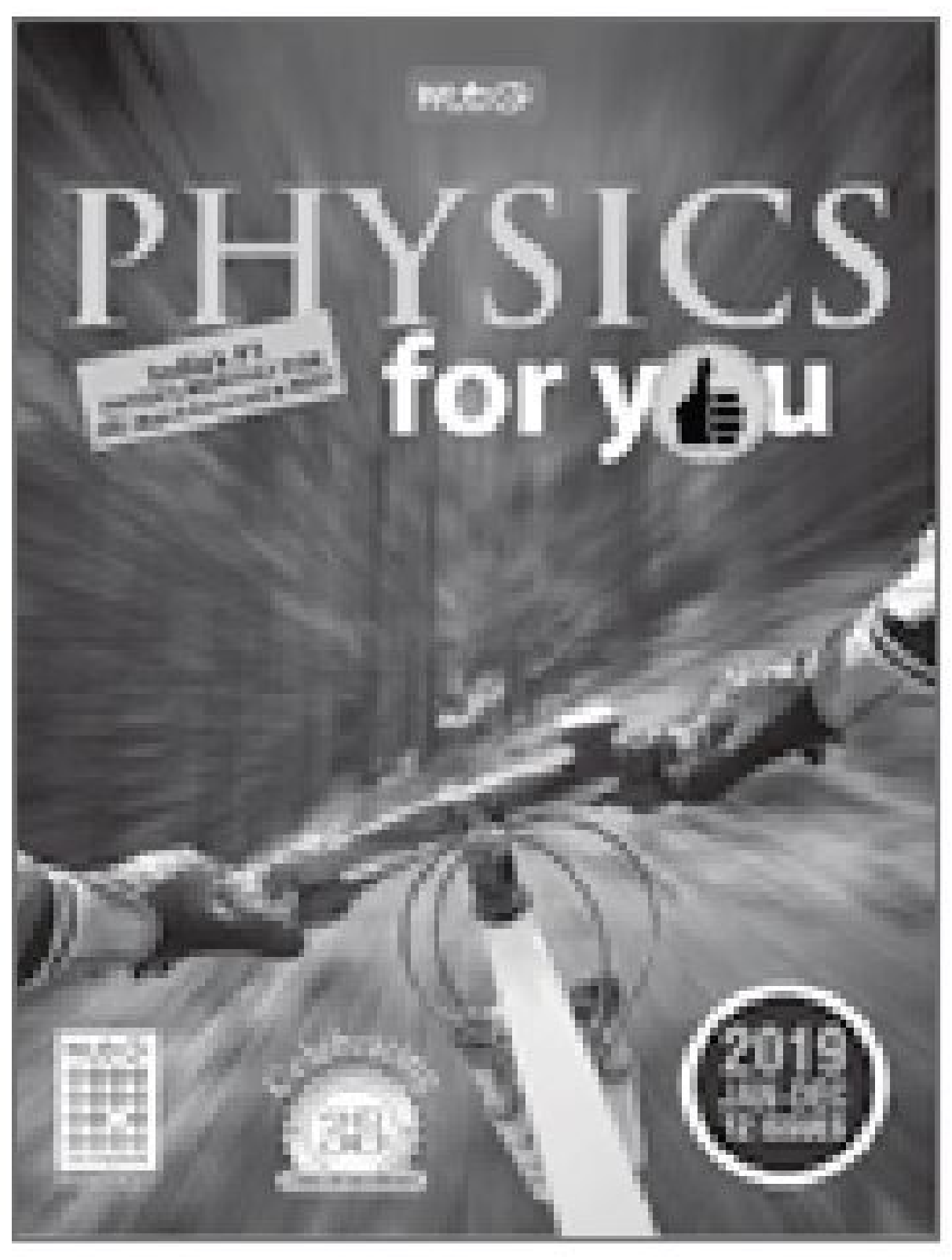
$\Rightarrow n(A \cap B) + n(B \cap C) + n(C \cap A) = 76 - 58 = 18$

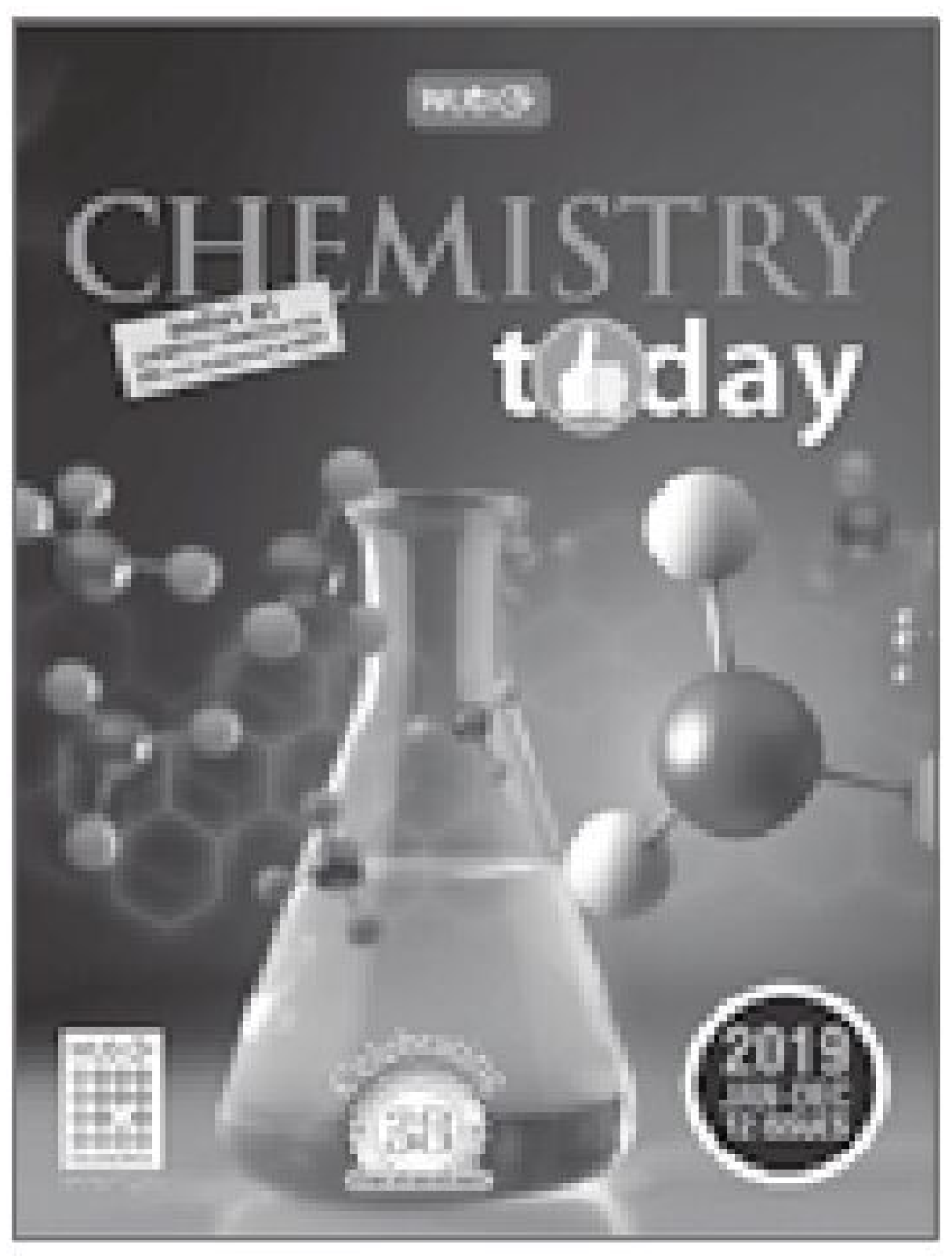
$\Rightarrow (a + 3) + (b + 3) + (c + 3) = 18$

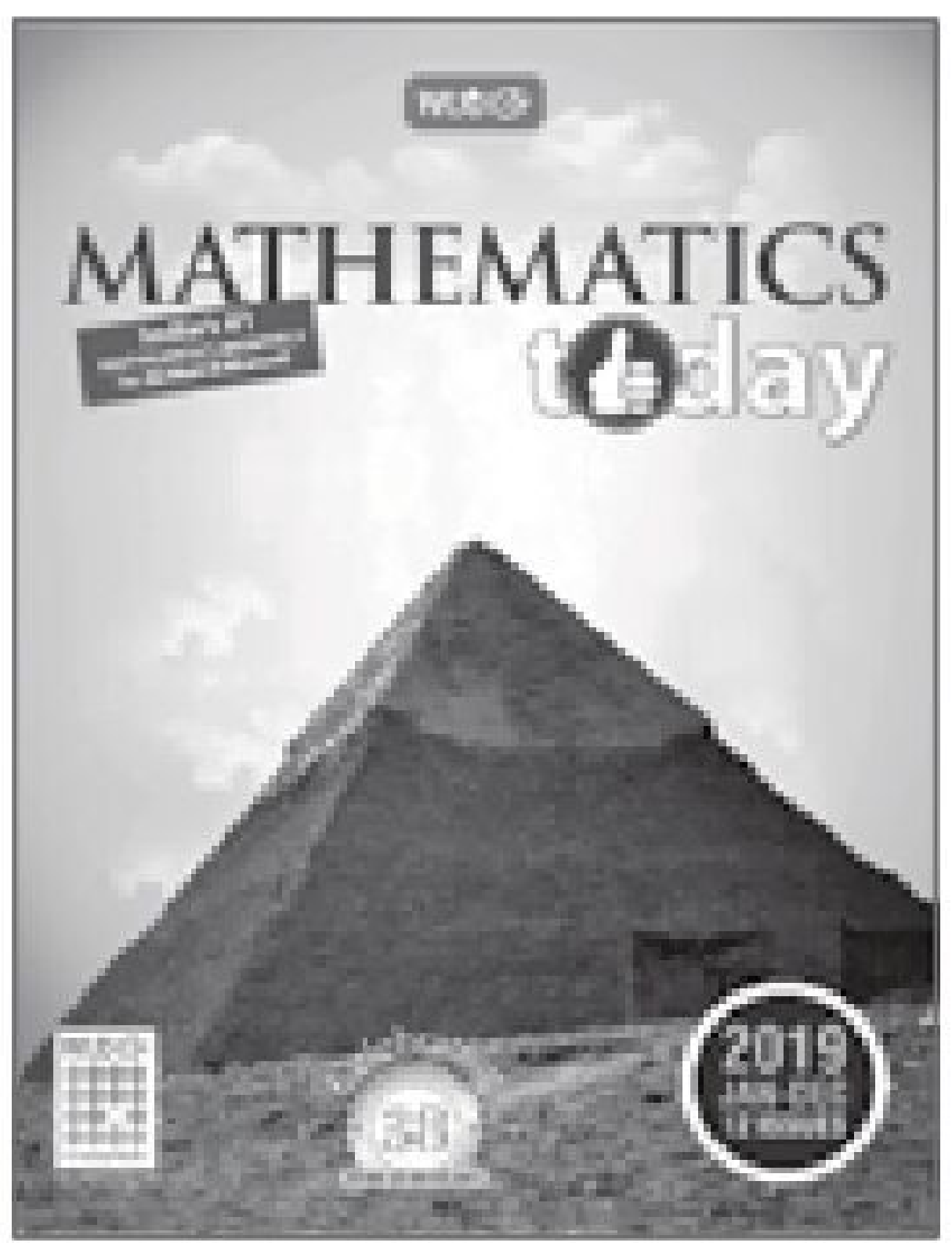
$\Rightarrow a + b + c = 9$

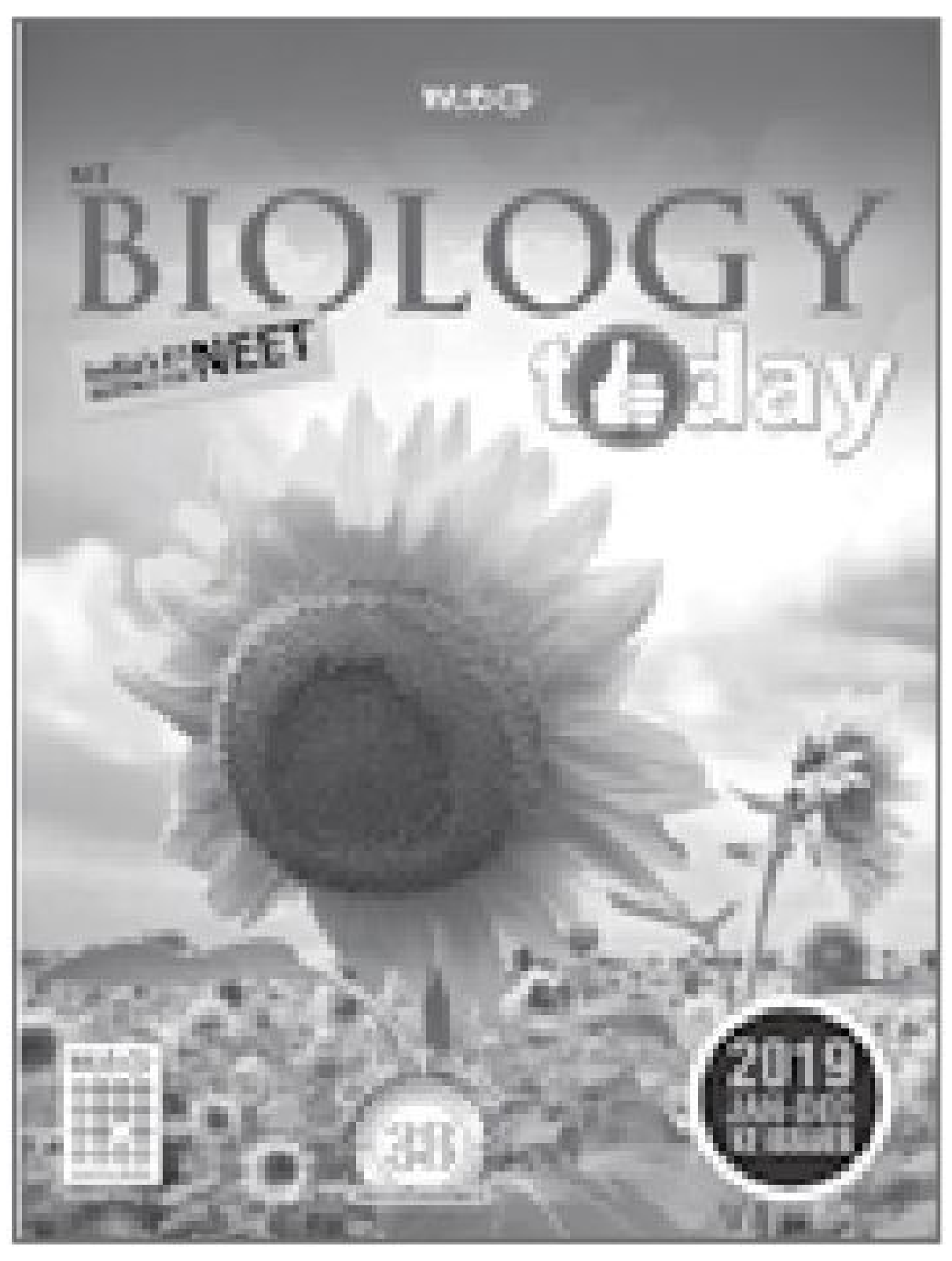
Hence, 9 men received medals in exactly two of the three sports.

◆ ◆









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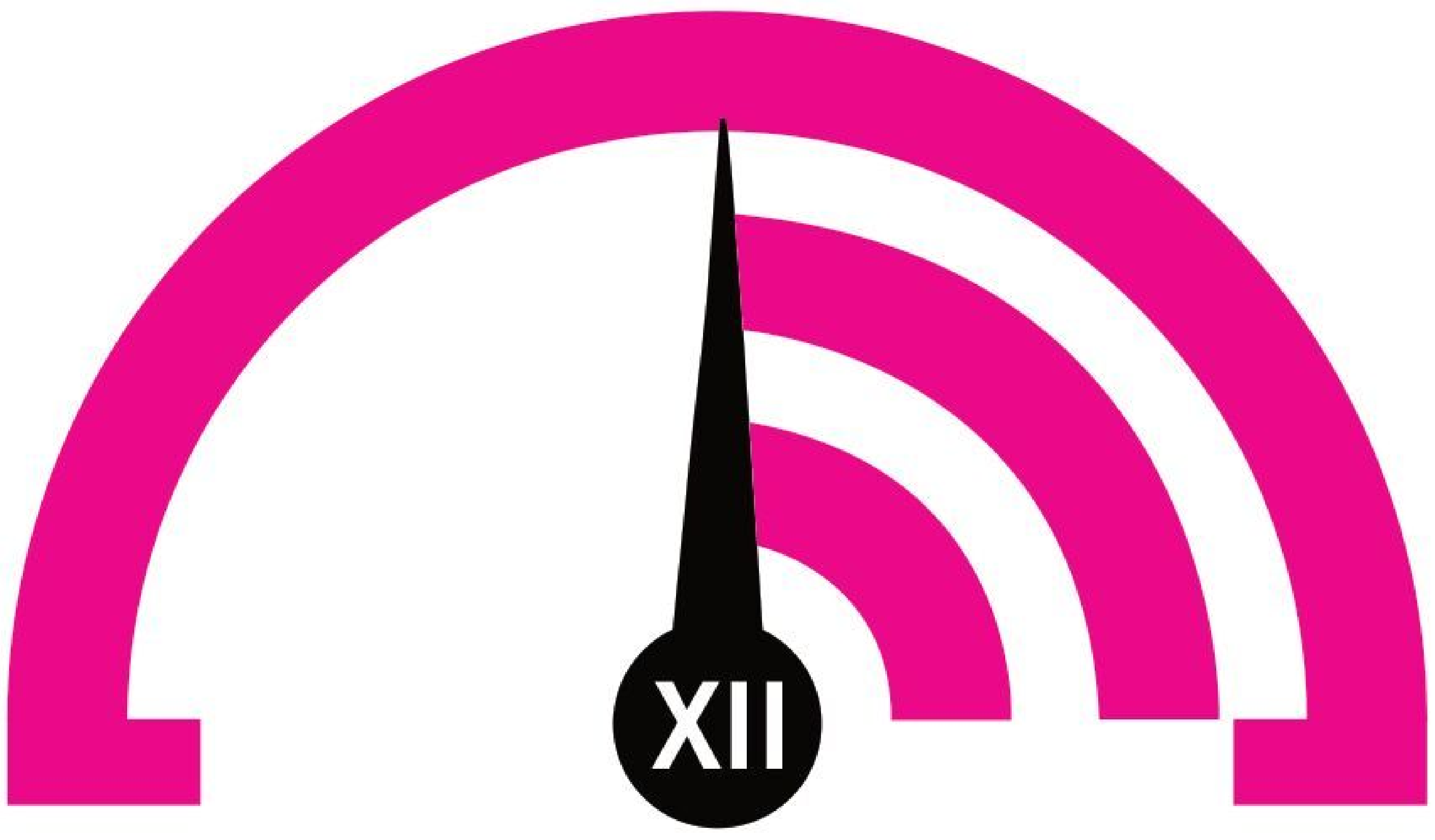

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 1 : Relations and Functions

Time Taken : 60 Min.

Only One Option Correct Type

1. Let $A = \{a, b, c\}$ and $f = \{(a, c), (b, a), (c, b)\}$ be a function from A to A. Then f^{-1} is

(a) $\{(c, a), (a, b), (b, c)\}$
(b) $\{(a, a), (b, b), (c, c)\}$
(c) $\{(a, c), (b, a), (c, b)\}$
(d) none of these

2. Let $f(x)=\begin{cases}1+x, & 0\le x\le 2\\ 3-x, & 2<x\le 3\end{cases}$, then $fof(x)$ =

(a) $\begin{cases}2+x, & 0\le x\le 1\\ 2-x, & 1<x\le 2\\ 4-x, & 2<x\le 3\end{cases}$ (b) $\begin{cases}2+x, & 0\le x\le 2\\ 4-x, & 2<x\le 3\end{cases}$

(c) $\begin{cases}2+x, & 0\le x\le 2\\ 2-x, & 2<x\le 3\end{cases}$ (d) none of these

3. Let $f: R \to R$ defined by $f(x)=\dfrac{e^{x^2}-e^{-x^2}}{e^{x^2}+e^{-x^2}}$, then

(a) $f(x)$ is one-one but not onto
(b) $f(x)$ is neither one-one nor onto
(c) $f(x)$ is many one but onto
(d) $f(x)$ is one-one and onto

4. The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which the function $f(x)=4^{-x^2}+\cos^{-1}\left(\frac{x}{2}-1\right)+\log(\cos x)$ is defined, is

(a) $[0, \pi]$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(c) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) $\left[0, \frac{\pi}{2}\right)$

5. Let f be a real valued function defined by

$f(x)=\dfrac{e^x-e^{-|x|}}{e^x+e^{|x|}}$, then range of f is

(a) R (b) $[0, 1]$
(c) $[0, 1)$ (d) $[0, 1/2)$

6. Let the function $f(x) = x^2 + x + \sin x - \cos x + \log(1 + |x|)$ be defined over the interval $[0, 1]$. The odd extension of $f(x)$ in the interval $[-1, 0]$ is

(a) $x^2 + x + \sin x + \cos x - \log(1 + |x|)$
(b) $-x^2 + x + \sin x + \cos x - \log(1 + |x|)$
(c) $-x^2 + x + \sin x - \cos x + \log(1 + |x|)$
(d) none of these

One or More than One Option(s) Correct Type

7. If $f(x)=\dfrac{x}{x^2+1}$ and $f(A)=\left\{y: -\frac{1}{2}\le y<0\right\}$, then set A is

(a) $[-1, 0)$ (b) $(-\infty, -1]$
(c) $(-\infty, 0)$ (d) $(-\infty, \infty)$

8. If the function $f: R \to R$ be such that $f(x) = x - [x]$, where $[\cdot]$ denotes the greatest integer function, then $f^{-1}(x)$ is

(a) $\dfrac{1}{x-[x]}$ (b) $[x] - x$
(c) not defined (d) none of these

9. Let $f(x) = [x]^2 + [x + 1] - 3$, where $[x] \le x$. Then

(a) $f(x)$ is a many-one and into function
(b) $f(x) = 0$ for infinite number of values of x
(c) $f(x) = 0$ for only two real values
(d) none of these

10. If $f(x) = \cos([\pi^2]x) + \cos([-\pi^2]\,x)$, where $[x]$ stands for the greatest integer function, then

(a) $f\left(\frac{\pi}{2}\right) = -1$ (b) $f(\pi) = 1$

(c) $f(-\pi) = 0$ (d) $f\left(\frac{\pi}{4}\right) = 1$

11. Let $A = \{a, b, c\}$ and $R = \{(a, a), (b, b), (c, c), (b, c)\}$ be a relation on A. Here, R is

(a) reflexive
(b) symmetric
(c) anti-symmetric
(d) transitive

12. The domain of $f(x)$ is $(0, 1)$, therefore domain of $f(e^x) + f(\ln|x|)$ is

(a) $(-1, e)$ (b) $(1, e)$
(c) $(-e, -1)$ (d) $(-e, 1)$

13. The possible values of 'a' for which the function $f(x) = e^{x-[x]} + \cos ax$ (where $[\cdot]$ denotes the greatest integer function) is periodic with finite fundamental period is/are

(a) π (b) 2π
(c) 3π (d) 1

Comprehension Type

Let $f(x)=\dfrac{x^2-x+1}{x^2+x+1}$ for $x \in R$, $g(x) = e^x$ for $x \in R$ and

$h(x) = \tan x$ for $-\dfrac{\pi}{2}<x<\dfrac{\pi}{2}.$

14. Range of the function foh is

(a) $\left(\frac{1}{3}, 3\right)$ (b) $\left[\frac{1}{3}, 3\right]$

(c) $\left[\frac{1}{3}, 1\right]$ (d) $[1, 3]$

15. Range of the function goh is

(a) R (b) $[0, \infty)$
(c) $(-\infty, 0]$ (d) $(0, \infty)$

Matrix Match Type

16. Match the following columns :

	Column I		**Column II**
(A)	Let $f(x) =$ max $\{1 + \sin x, 1, 1 - \cos x\}$, $x \in [0, 2\pi]$ and $g(x) = \max\{1, \lvert x - 1\rvert\}$, $x \in R$, then	(p)	$g(f(1)) = 1$
(B)	Let $f(x)=\ln\left(\dfrac{1+x}{1-x}\right)$ $\forall\, x \in (-1, 1)$ and $g(x)=\left(\dfrac{3x+x^3}{1+3x^2}\right)$, then	(q)	$f(g(0)) = 0$
(C)	Let $f(x) = 1 + x^2$ and $g(x) = x - x^2$, then	(r)	$f(g(0)) = 1$
		(s)	$g(f(0)) = 2$

(a) (A) → q ; (B) → r ; (C) → s
(b) (A) → p ; (B) → r ; (C) → s
(c) (A) → p ; (B) → q ; (C) → r
(d) (A) → q ; (B) → q ; (C) → r

Integer Answer Type

17. If $f\left(2x^2+\dfrac{y^2}{8}, 2x^2-\dfrac{y^2}{8}\right) = xy$, then $f(60, 48) + f(80, 48) + f(13, 5) = M$. Find the sum of the digits of M.

18. The range of the function $f(x)=\sqrt{(x-6)}+\sqrt{(12-x)}$ is an interval of length $\sqrt{\lambda}-\sqrt{\mu}$, then $\lambda - \mu$ must be _____.

19. Let $n(A) = 4$ and $n(B) = 6$, then the number of one-one functions from A to B is $(6\sqrt{\lambda})^2$, find $\dfrac{\lambda}{2}$.

20. Total number of solutions of the equations $2^x|2 - |x|| = 1$ are _____.

◆◆

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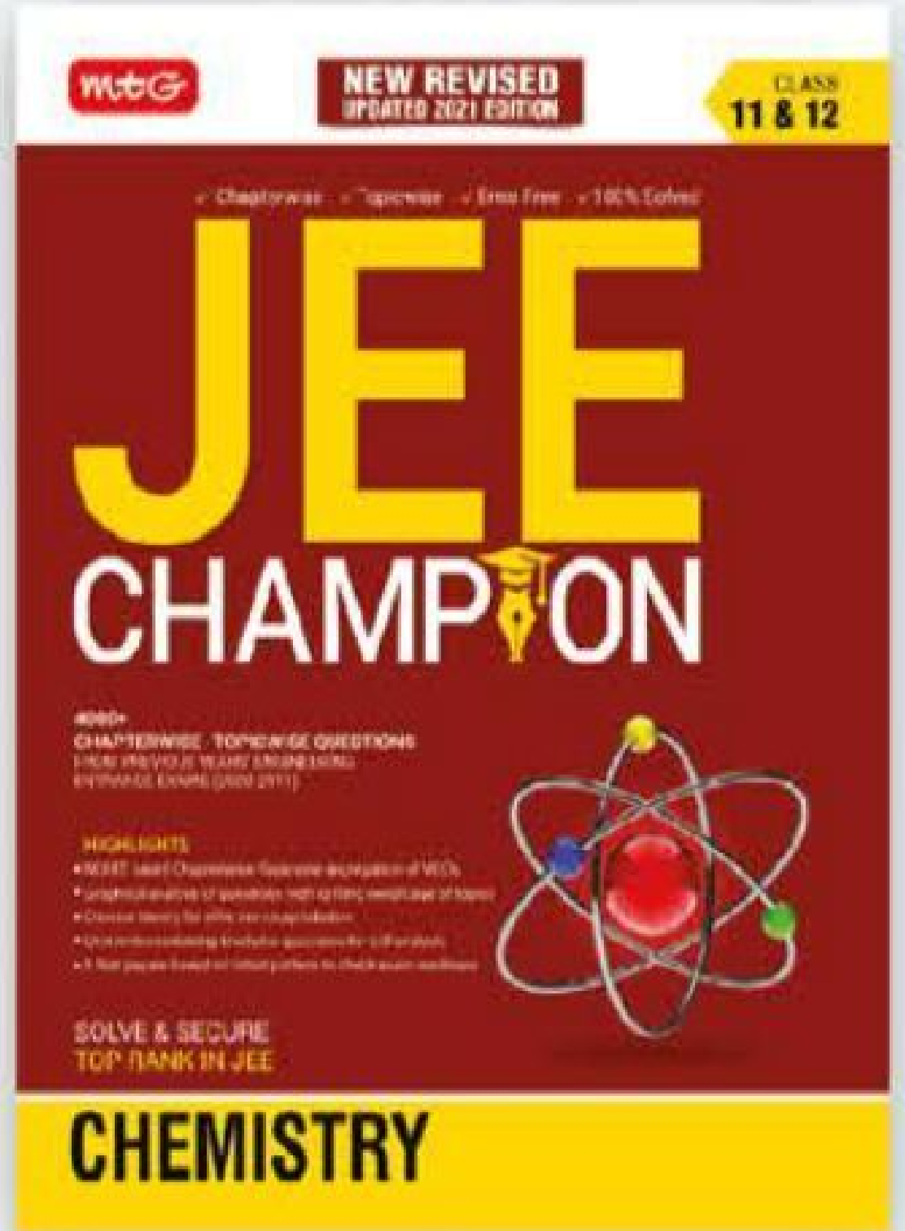
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